A REVIEW OF SELF-SIMILAR MOTION OF SELF-GRAVITATING STRONG SHOCKS IN A CONDUCTING MEDIUM WITH RADIATIVE HEAT FLUX

THESIS PRESENTED

BY

NEETA AGARWAL

TO FULFIL PARTIAL REQUIREMENT FOR THE DEGREE OF MASTER OF PHILOSOPHY OF

BUNDELKHAND UNIVERSITY
JHANSI
(INDIA)

DEPARTMENT OF
MATHEMATICS & STATISTICS

SUPERVISOR
DR. V. K. SINGH

1989

CERTIFICATE

THIS IS TO CERTIFY that work embodied in the THESIS entitled " A REVIEW OF SELF-SIMILAR MOTION OF SELF-GRAVITATING STRONG SHOCKS IN A CONDUCTING MEDIUM WITH RADIATIVE HEAT FLUX " being submitted by Neeta Agarwal, to fulfil partial requirement for the Degree of M.Phil. of Bundelkhand University, Jhansi, U.P., is upto the mark both in its academic contents and the quality of presentation.

I further certify that this work has been done by her under my supervision and guidance.

Vijai Krishna Singh

Dated, Jhansi
December 26th, 1989

(Dr. V.K.Singh)
Lecturer,
Deptt. of Mathematics & Statis
Bundelkhand University,
Jhansi.

P R E F A C E

The present work is an outcome of the studies made by me under the supervision of Dr.V.K.Singh, Lecturer in the department of Mathematics and Statistics, Bundelkhand University, Jhansi and is being submitted for the award of M.Phil. Degree in Mathematics.

This Thesis has been divided into three Chapters and each Chapter has been sub-divided into sections. The Chapter-I gives an idea about the continuum concept of fluid; Basic equations governing the flow of a conducting fluid; Shock-waves, their existence and conditions; Radiation phenomenon; Similarity principle for a self-similar flow and the concept of the self-gravitation. Chapter II and III consist of the solutions of the two research papers which are enclosed at the end of the work. A list of notation is given in the starting which is used throughout the work. At the end of the thesis references are given which include research papers and texts which have been consulted during the preparation of this work.

I am under great obligation to my distinguished teacher, Dr.V.K.Singh for his extremely valuable suggestions, guidance and keen interest throughout the progress of my work.

I am grateful to Dr.P.N.Shrivastava, Reader and Head, Department of Mathematics and Statistics, Bundelkhand University, for his valuable comments and useful suggestions in cours of my dissertation work.

I am thankful to Dr.V.K.Saighal of the department for

their continuous encouragement and interest in my work.

I would like to express my sincere gratitude to my parents who have always been a source of encouragement and inspiration in my life.

My thanks are also due to my class mates for many useful discussions and for their assistance in many ways.

Finally, I am thankful to Mr. V.S.Srivastava who typed this manuscript very interestingly.

Neeta Agarwal.

(Neeta Agarwal)

Dated, Jhansi
December 25th, 1989

CONTENTS

The solutions of the following two problems are given in Chapter II and III, respectively.

B.G.Verma and V.K.Singh
Department of Mathematics, University of Gorakhpur, India

Astrophysics and space science 124(1986) 53-62

LIST OF NOTATION

p	pressure
ρ	density
T	temperature
$\bar{v}$	velocity vector
V	volume
$\bar{r}$	position vector
t	time
γ	specific heat ration
Q	heat added to unit mass
h	enthalpy per unit mass
g	gravitational acceleration
u, v, w	Cartesion components of velocity
x, y, z	Cartesion coordinates
δ_{ij}	Kronecker delta
ϵ_{ijk}	permutation symbol
$\bar{F}$	force vector
c	velocity of light
a	velocity of sound
E	energy
Γ	gas constant
r	radial distance
G	gravitational constant
η	similarity variable
C_p	specific heat at constant pressure
C_v	specific heat at constant volume

$\bar{H}$	magnetic field intensity
$\bar{E}$	electric field intensity
$\bar{B}$	magnetic induction vector
$\bar{D}$	electric displacement vector
$\bar{J}$	current density vector
σ	conductivity of the medium
μ	permeability of the medium
q	charge density of the medium
ϵ	dielectric constant of the medium
$\mathcal{E}$	internal energy per unit mass

CHAPTER-I

1. INTRODUCTION

The study of 'Fluid Dynamics' is directed to the behaviour of a fluid in motion. The liquid and gas states are referred to generally as fluids. Some of the notable examples of application are : flow of water through pipes; motion of an aircrafts or a missiles in the atmosphere. The study also yields the methods and devices for the measurement of various parameters, e.g. the pressure and velocity in a fluid at rest or in motion.

The first notable works on fluids appeared in the seventeenth century. In 1687, Newton in his book 'Principia', deals with the influx of a fluid, the resistance of a fluid and the resistance of projected bodies and the wave motion. Daniel Bernoulli, in 1738 determined the relation between pressure and velocity and formalized this in his theorem. In 1743 his son John Bernoulli applied the momentum principle to infinitesimal elements. Euler realized the significance of the Bernoullis' work and used it as the basis, setting down the fundamental equations of motion and equation of continuity for an ideal inviscid fluid in 1755. An alternate form of these equations was given by Lagrange, in 1781 and 1789. Equations including viscosity were derived by Navier in 1822 and Stokes in 1845.

In 1858 Helmholtz published a paper on vortex motion and in 1868 another on free stream line potential flow.

Near the end of the nineteenth century, fluid flow

itself began to be extensively observed and investigated. The modern trend in fluid dynamics is to investigate the flow of electrically conducting fluid at very high temperature.

At a microscopic level, the three states of a given substance are different because of the difference in the intermolecular distance. In many cases at normal conditions the molecular distance of a fluid are less than the minutest of any physical dimensions of practical interest. As a result, we interested in the statistical average properties and the behaviour of large numbers of molecules, and not in that of individual molecules. That is, macroscopic and not microscopic, properties are of interest. In Fluid dynamics as individual molecules are not being considered, the fluid can be regarded as a continuous medium and so the physical quantities such as mass, momentum etc. of the fluid contained in a very small volume are regarded as being spread uniformly throughout that volume.

In dealing with the gases at very low pressure, as in the upper atmosphere, or at very high temperatures such as in a plasma, the continuum concept of fluid dynamics must be violated and the study that has to be based on the behaviour of individual molecules (i.e. on the macroscopic approach)[1].

In our studies we frequently refer to a 'small element of fluid' which is always supposed so large that it still contains a very large number of molecules, as fluid is a continuous medium. So when we take of an

infinitesimal element of volume we always mean that which are physically infinite small. Such an element is called a fluid particle.

In continuum dynamics we assum that the macroscopic fluid properties, for example mean density, mean pressure, vary continuously with (a) the size of element of fluid considered (b) the position in the fluid, and (c) the time. In (a), the variation becomes negligible as the element is physically very small. Thus, fluid properties density, pressure and velocity are expressed as continuous functions of position and time only. On this basis, it is possible to establish equations governing the motion of a fluid, which are independent of the nature of the particle structure. So gases and liquids may be treated together.

There are two distinct methods of specifying the flow field [2] -

(i) <u>LAGRANGIAN METHOD</u>: In this method, the flow variables (velocity, pressure and density) of a selected fluid element or particle are described. If $\bar{r}_0$ is the position of the center of mass of the fluid element at time t_0 then the basic flow quantity in the Lagrangian description is the velocity $\bar{v}$ (r_0, t). This method is also referred as 'individual time rate of change'.

(ii) <u>EULERIAN METHOD</u>: In this method, the flow quantities are described at all points of space occupied by the fluid at all times i.e. flow quantities are defined as the function of position in space ($\bar{r}$) and time (t). The basic flow quantity is the vector velocity $\bar{v}$ (r,t). This method corresponds to ' local time rate of change '.

with some property of the fluid (it could be density, velocity etc.). Suppose fluid particle has the position $P(x,y,z)$ or $P(\bar{r})$ at time t. Keeping this point fixed the change in f is during the interval of time δt is,

$$f(x,y,z,t+\delta t) - f(x,y,z,t).$$

Hence the 'local time rate of change' is given by,

$$\frac{\partial f}{\partial t} = \lim_{\delta t \to 0} \frac{f(x,y,z,t+\delta t) - f(x,y,z,t)}{\delta t}$$

As the point P is fixed the local time differential operator $\frac{\partial f}{\partial t}$ is not carried along by the moving fluid.

Now let at time $t+\delta t$ the fluid particle, which was at the position (x,y,z) originally, is in the position $(x + u\delta t,\ y + v\delta t, z + w\delta t)$ where u,v, and w be the velocity components at the position at time t. The corresponding change of f is given by,

$$f(x + u\delta t,\ y + u\delta t,\ z + w\delta t,\ t + \delta t) - f(x,y,z,t)$$

or $\quad f(\bar{r} + \delta\bar{r}, t + \delta t) - f(\bar{r}, t)$

and rate of change is,

$$\frac{df}{dt} = \lim_{\delta t \to 0} \frac{f(\bar{r} + \delta\bar{r},\ t + \delta t) - f(\bar{r},t)}{\delta t}$$

This gives the 'individual time rate of change'. As the point P is moving it gives the rate of change which is carried along by the moving fluid.

Since,

$$f = f(x,y,z,t)$$

$$\therefore\ df = \frac{\partial f}{\partial t}\,dx + \frac{\partial f}{\partial y}\,dy + \frac{\partial f}{\partial z}\,dz + \frac{\partial f}{\partial t}\,dt$$

so,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

If $\bar{v} = [u,v,w]$ be the velocity of the fluid particle at P and $\frac{dx}{dt} = u$, $\quad \frac{dy}{dt} = v$, $\quad \frac{dz}{dt} = w$

then,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} u + \frac{\partial f}{\partial y} v + \frac{\partial f}{\partial z} w$$

$$= \frac{\partial f}{\partial t} + \left(i \frac{\partial f}{\partial x} + j \frac{\partial f}{\partial y} + k \frac{\partial f}{\partial z}\right) \cdot (ui + vj + wk)$$

or,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + (\bar{v} \cdot \nabla)\, f$$

where $\bar{v} = ui + vj + wk$

The first term on the right hand side represents the local rate of change of f, and the second term the convective rate of change.

If $\bar{v}$ is introduced for f in the above equation then the total derivative of velocity with respect to time is,

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla)\, \bar{v}$$

In cartesion rectangular coordinates

$$(\bar{v} \cdot \nabla) = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

It is this term in the Euler relation for acceleration which is responsible for the non-linearities in the equations of motion of fluid dynamics.

2. FUNDAMENTAL EQUATIONS

In order to study the details of the fluid flow, we wish to find the density distribution, pressure distribution, velocity distribution, states of the fluid, etc., at all points of space occupied by the fluid at all times. Hence a knowledge of three velocity components (u,v,w), the temperature T, the pressure p, the density ρ, etc., of the fluid which are functions of position in space ($\bar{r}$) i.e. (x,y,z) and the time t, is needed. We obtain relations connecting these unknowns and which would explain the particular problem of the fluid motion. We call these relations as fundamental equations.

The flow of a compressible, non-heat conducting and inviscid perfect fluid is described by the four physical variables pressure p, density ρ, vector velocity $\bar{v}$, and temperature T. So there must be four fundamental equations to find these variables. These are [3],

Equation of continuity,

$$\frac{\partial \rho}{\partial t} + \text{div}\,(\rho \bar{v}) = 0$$

or

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \quad \text{in one dimension}$$

Equations of motion,

$$\frac{D\bar{v}}{Dt} = \frac{\partial \bar{v}}{\partial t} + (\bar{v}.\nabla)\bar{v} = -\frac{1}{\rho}\nabla p$$

When the external force is negligible.

Energy equation,

$$\frac{DQ}{Dt} = \frac{Dh}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt}$$

for the adiabatic flow,

$$\rho\frac{Dh}{Dt} - \frac{Dp}{Dt} = 0$$

and equation of state for perfect gas,

$$p = \rho RT$$

when there is no intermolecular attractions.

Now we consider the behaviour of the fluid flow at high temperature, where some of the fluids become conducting and the interaction between motions of the conducting fluids and variations in electromagnetic fields may not be negligible. To describe this interaction phenomena of the flow field with the electromagnetic field we find the velocity vector $\bar{v}$, pressure p, density ρ , temperature T, and the magnetic field vector $\bar{H}$, which are the functions of the spatial coordinates (x,y,z) and the time t. To calculate these unknowns we must find the relations which are the fundamental equations for a conducting compressible fluid. In these equations there are coupling terms between the electromagnetic and fluid dynamics phenomena. We shall derive and discuss these equations([4], [5], [6])

EQUATION OF CONTINUITY :

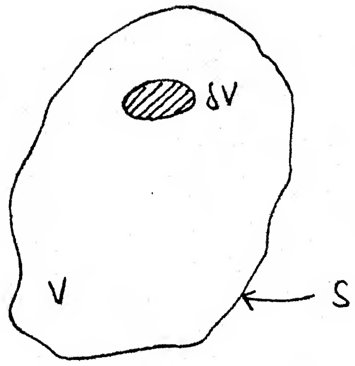
This equation simply expresses the law of conservation of mass.

The quantity of fluid entering a certain volume in space must be balanced by that quantity leaving i.e. matter is neither created nor destroyed.

We now formulate this principle mathematically.

Let V be any arbitrary volume fixed in space, bounded by a surface S, and containing a fluid of density ρ. The volume element δV is small so that ρ can be regar-

ded as constant through it.



The mass of the fluid with in the volume V is $\int_V \rho \, dV$

The rate of generation of the fluid with in the volume is,

$$\frac{\partial}{\partial t}\int_V \rho \, dV = \int_V \frac{\partial \rho}{\partial t} dV$$

For $\rho \frac{\partial}{\partial t}(dV) = \rho \, d(\frac{\partial V}{\partial t}) = \rho \cdot 0 = 0$, as volume is constant with respect to time.

If the volume V occupied by a moving fluid, the fluid enters V through parts of its boundary surface S and leaves through another part.

Let $\hat{n}$ be a unit outward normal vector drawn on the surface element dS. The normal velocity is $\hat{n}.\bar{v}$. The total outward flux is,

$$\int_S \rho(\hat{n}.\bar{v})\, dS = \int_S \hat{n}.(\rho\bar{v})\, dS$$

Using Gauss's theorem,

$$= \int_V \nabla \cdot (\rho\bar{v})\, dV$$

The sum of the net outward convection of mass plus the rate of generation, of the fluid with in the volume must be zero.

$$\int_V \nabla \cdot (\rho\bar{v})\, dV + \int_V \frac{\partial \rho}{\partial t} dV = 0$$

or

$$\int_V \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\bar{v})\right] dV = 0$$

Since this is true for arbitrary elementary volumes,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0 \qquad (1.1)$$

or,

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho v_i) = 0 \qquad i = 1,2,3$$

The Maxwell's electromagnetic equations for a conducting medium are,

$$\text{curl } \bar{E} = - \frac{\partial \bar{B}}{\partial t} \quad \text{where } \bar{B} = \mu \bar{H}$$

$$\text{curl } \bar{H} = 4\pi \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\text{div } \bar{B} = 0$$

and,

$$\text{div } \bar{D} = 4\pi q$$

Differentiating the fourth Maxwell's equation

$$\text{div } \left(\frac{\partial \bar{D}}{\partial t}\right) = 4\pi \frac{\partial q}{\partial t}$$

using the second Maxwell's equation it becomes,

$$\text{div } (\text{curl } \bar{H} - 4\pi \bar{J}) = 4\pi \frac{\partial q}{\partial t}$$

Third Maxwell's equation is

$$\text{div } \bar{H} = 0$$

we have,

$$- 4\pi \text{ div } \bar{J} = 4\pi \frac{\partial q}{\partial t}$$

or,

$$\frac{\partial q}{\partial t} + \text{div } \bar{J} = 0$$

If the charged particle moves with velocity $\bar{v}$

$$\bar{J} = q\bar{v} \qquad \text{then,}$$

$$\frac{\partial q}{\partial t} + \text{div } (q \bar{v}) = 0$$

This is the equation of continuity for the electric charge moving under the effect of magnetic field which is similar to the equation (1.1).

EQUATION OF STATE:

The electrodynamical state relation is a simple relation between the current density $\bar{J}$, fields and fluid motion. Or, the Ohm's law is,

$$\bar{J} = \sigma \left\{ \bar{E} + \mu \bar{v} \times \bar{H} \right\} + q \bar{v}$$

Where σ is the electrical conductivity and $q\bar{v}$ is the current depending on the motion of the net charge q.

The state of a compressible fluid is defined by the pressure p, the entsopy S, the internal energy $\mathcal{E}$, the absolute temperature T, the mass density ρ and the specific volume V.

An equation of state is formed by expressing any one of the thermodynamic variables in terms of the other two quantities mentioned above.

For example,

$$p = p(\rho, T)$$

or, $$E = E(P, \rho)$$

For a perfect fluid (no intermolecular intractions), having negligible viscosity the equation is,

$$p = R \rho T$$

Where, R is the universal gas constant.

Assuming the process to be adiabatic and isentropic we have,

$$\mathcal{E} = C_v T$$

$$\mathcal{E} = \frac{p}{R\rho} C_v \qquad \text{as} \quad p = \rho RT$$

Also, $$C_p - C_v = R$$

or, $$\frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

or, $$\gamma - 1 = \frac{R}{C_v}$$

$$\therefore \quad c_v = \frac{R}{\gamma - 1}$$

so,

$$\varepsilon = \frac{p}{\rho(\gamma - 1)}$$

This is the caloric equation of state of the medium. ε is the internal energy per unit mass.

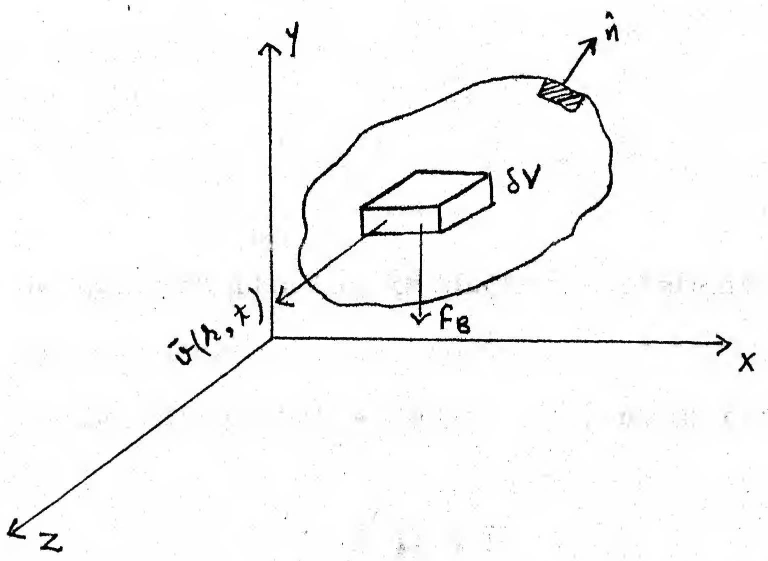
When a fluid element changes its state isentropically, the state equation is,

$$\frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = 0$$

EQUATIONSOF MOTION:

This equation express the laws of conservation of momentum in the fluid. To obtain equations deriving this we proceed as following:-

We consider a fluid mass in motion at time t occupying a volume V and bounded by a surface S. Let δV be a small element of volume. If the fluid is of density ρ, the mass of the element is $\rho \delta V$ and it moves with velocity $\bar{v}(\bar{r}, t)$



The inertial force on the element is $\rho\,\delta V\left(\frac{D\bar{v}}{Dt}\right)$. This force equals the rate of change of linear momentum of the element. For the whole fluid mass,

$$F_I = \iiint \frac{D\bar{v}}{Dt}\,\rho\,\delta V$$

The force on the body is the sum of body and pressure force.

Consider a surface element δS and that $\hat{n}$ be the unit outward normal vector to δS. Let p is the fluid pressure at any point on the element δS. Then the force on it due to the fluid outside the particle is,

$$p(-\hat{n})\,\delta S \qquad \text{(For pressure acts along inward)}$$

Hence the total pressure force is,

$$-\iint_S \hat{n} p\, dS$$

using Gauss's theorem,

$$F_p = -\iiint_V \text{grad } p\, dV$$

When the fluid moves in an electric and magnetic field, the body force F_B consists of three parts: gravitational, electric and magnetic.

The gravitational body force on the element is $\bar{g}\,\rho\,\delta V$ where $\bar{g}$ is the gravitational acceleration.

The electric body force on it due to an electric field of intensity $\bar{E}$ would be $\bar{E}\, q\, \delta V$.

The current flowing through the element is, $\bar{i} = \bar{J}\,\delta A$. Where δA be normal cross - section of fluid element.

Hence using Biot - Savart law, magnetic force in the element is,

$$\bar{i}\,\delta l \times \bar{B}$$

$$= \bar{J}\,\delta A\,\delta l \times \bar{B}$$

$$= (\bar{J} \times \bar{B})\,\delta V$$

$$= \mu(\bar{J} \times \bar{H})\,\delta V$$

Hence the total body force F_B on the fluid mass is,

$$F_B = \iiint_V (\rho\bar{g} + \mu\bar{J} \times \bar{H} + q\bar{E})\,dV$$

Now,

$$F_I = F_B + F_p$$

So,

$$\iiint_V \left\{-\rho\frac{D\bar{v}}{Dt} + (\rho\bar{g} + \mu\bar{J} \times \bar{H} + q\bar{E}) - \text{grad}\,p\right\} dV = 0$$

The above equation is true for arbitrary elementary volumes and so,

$$\rho\frac{D\bar{v}}{Dt} = -\text{grad}\,p + \rho\bar{g} + \mu\bar{J} \times \bar{H} + q\bar{E}$$

This is the general equation of motion for a conducting fluid. Where Eulerian derivative is,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\bar{v}.\nabla)$$

The term involving $\bar{E}$ can be neglected as compare to the term involving magnetic field $\bar{H}$. As the order of magnitude of $q\bar{E}$ is very-very small as compare to the order of magnitude of $\mu\bar{J} \times \bar{H}$ i.e.

$$O(|q\bar{E}|) << O(|\mu\bar{J} \times \bar{H}|)$$

So the modified form of equation of motion is,

$$\rho\frac{D\bar{v}}{Dt} = -\text{grad}\,p + \rho\bar{g} + \mu\bar{J} \times \bar{H}$$

We know, $\quad \text{curl}\,\bar{H} = 4\pi\bar{J} + \epsilon\frac{\partial\bar{E}}{\partial t}$

The term $\epsilon \frac{\partial \bar{E}}{\partial t}$ can be neglected as compare to curl $\bar{H}$ as,

$$o\left(\left| \epsilon \frac{\partial \bar{E}}{\partial t} \right| \right) << o\left(\left| \text{Curl } \bar{H} \right| \right)$$

so,

$$\text{Curl } \bar{H} = 4\pi \bar{J}$$

Then the magnetic force is,

$$\mu \bar{J} \times \bar{H} = \frac{\mu}{4\pi} \text{ Curl } \bar{H} \times \bar{H}$$

or

$$= - \frac{\mu}{4\pi} \bar{H} \times \text{Curl } \bar{H}$$

Since, (using vector identity)

$$\bar{H} \times \text{Curl } \bar{H} = \text{grad } \left(\frac{H^2}{2}\right) - (\bar{H} . \nabla)\, \bar{H}$$

Then,

$$\mu \bar{J} \times \bar{H} = - \text{grad } \left(\frac{\mu H^2}{8\pi} \right) + \frac{\mu}{4\pi} (\bar{H} . \nabla)\, \bar{H}$$

So equations of motion can be written as,

$$\rho \frac{D\bar{v}}{Dt} = - \text{grad } p + \rho \bar{g} - \text{grad } \left(\frac{\mu H^2}{8\pi} \right) + \frac{\mu}{4\pi} (\bar{H} . \nabla) \bar{H}$$

In summation convention form, it can be written as,

$$\frac{\partial}{\partial t} (\rho v_j) + v_j \frac{\partial}{\partial x_i} (\rho v_i) + \left(v_i \frac{\partial}{\partial x_i}\right) v_j =$$

$$= - \frac{\partial}{\partial x_i} \left(p + \frac{\mu H^2}{8\pi}\right) \delta_{ij} + \frac{\mu}{4\pi} \left(H_i \frac{\partial}{\partial x_i}\right) H_j$$

where

i is the dummy index

j is the free index

and $i, j = 1, 2, 3$

EQUATION FOR THE VARIATION OF THE MAGNETIC FIELD:

This equation expresses how the magnetic field vary or the time dependence of the magnetic field.

The Maxwell's electromagnetic field equations for a conducting medium are,

$$\text{div } \bar{E} = 4\pi q$$

$$\text{div } \bar{H} = 0$$

$$\text{Curl } \bar{E} = -\frac{1}{c}\frac{\partial \bar{H}}{\partial t}$$

and,
$$\text{Curl } \bar{H} = \frac{4\pi}{c}\bar{J} + \frac{1}{c}\frac{\partial \bar{E}}{\partial t}$$

The first two equations are derived by applying Gauss's theorem to a closed surface. Third equation is the mathematical formulation of the Faraday's law of induction and last equation describes how the magnetic field $\bar{H}$ depends on the conduction current $\bar{J}$ and the displacement current $(\frac{1}{c}\frac{\partial \bar{E}}{\partial t})$.

Since,

$$\text{Curl } \bar{H} = \frac{4\pi}{c}\bar{J} + \frac{1}{c}\frac{\partial \bar{E}}{\partial t}$$

By neglecting the term $\frac{1}{c}\frac{\partial \bar{E}}{\partial t}$ above equation reduces to,

$$\text{Curl } \bar{H} = \frac{4\pi}{c}\bar{J}$$

Taking Curl of this,

$$\text{Curl Curl } \bar{H} = \frac{4\pi}{c}\text{ Curl } \bar{J}$$

or,
$$\text{grad div } \bar{H} - \nabla^2 \bar{H} = \frac{4\pi}{c}\text{Curl}\left[\sigma\left(\bar{E} + \frac{1}{c}\bar{v} \times \bar{H}\right)\right]$$

where,
$$\bar{J} = \sigma(\bar{E} + \frac{1}{c}\bar{v} \times \bar{H})$$

Since, $\text{div } \bar{H} = 0$

so,

$$-\nabla^2 \bar{H} = \frac{4\pi\sigma}{c}\left[\text{Curl } \bar{E} + \frac{1}{c}\text{ Curl } (\bar{v} \times \bar{H})\right]$$

Also, $$\text{Curl } \bar{E} = -\frac{1}{c}\frac{\partial \bar{H}}{\partial t}$$

$$-\nabla^2 \bar{H} = \frac{4\pi\sigma}{c^2}\left[-\frac{\partial \bar{H}}{\partial t} + \text{Curl } (\bar{v} \times \bar{H})\right]$$

or, $$\frac{\partial \bar{H}}{\partial t} = \text{Curl } (\bar{v} \times \bar{H}) + \frac{c^2}{4\pi\sigma}\nabla^2 \bar{H}$$

we may write,

$$\frac{\partial \bar{H}}{\partial t} = \text{Curl } (\bar{v} \times \bar{H}) + \eta \nabla^2 \bar{H}$$

where $\eta = \frac{c^2}{4\pi\sigma}$

or,

$$\frac{\partial \bar{H}}{\partial t} = (\bar{H}.\nabla)\bar{v} - (\bar{v}.\nabla)\bar{H} - \bar{H}(\nabla.\bar{v}) + \eta \nabla^2 \bar{H}$$

In summation convention form, it can be written as,

$$\frac{\partial}{\partial t}(H_j) = (H_i \frac{\partial}{\partial x_i}) v_j - (v_i \frac{\partial}{\partial x_i}) H_j - H_j(\frac{\partial}{\partial x_i} v_i) + \eta (\frac{\partial^2}{\partial x_i^2}) H_i$$

This is the magnetic field equation for a conducting compressible fluid.

For a fluid at rest it reduces to the diffusion equation,

$$\frac{\partial \bar{H}}{\partial t} = \eta \nabla^2 \bar{H}$$

η is called magnetic diffusivity. For an equation of this type the rate of decay of the magnetic field is very rapid.

And when the conductivity is so large that the term $\eta \nabla^2 \bar{H}$ can be neglected, the behaviour of the magnetic field is given by,

$$\frac{\partial \bar{H}}{\partial t} = \text{Curl } (\bar{v} \times \bar{H})$$

or

$$\frac{\partial \bar{H}}{\partial t} = (\bar{H}.\nabla)\,\bar{v} - (\bar{v}.\nabla)\,\bar{H} - \bar{H}\,(\nabla.\bar{v})$$

In this case the rate of decay of magnetic field is much less therefore this is the case of high conductivity.

EQUATION OF ENERGY:

The law of conservation of energy leads to another equation of fluid flow - the energy equation.

The law of conservation of energy is equivalent to the first law of thermodynamics, which states that, if a small quantity of heat is added to a simple system it is used up in changing the internal energy of the system and in the external work done by the system. Mathematically this law can be expressed as,

$$d\varepsilon = dQ - pd\left(\frac{1}{\rho}\right)$$

Where dQ is the heat added per unit mass, $d\varepsilon$ is the increase in internal energy per unit mass and $pd\left(\frac{1}{\rho}\right)$ is the amount of work done during the change of volume $d\left(\frac{1}{\rho}\right)$. The heat dQ is obtained by heat conduction.

For a perfect gas, the molecules have very negligible volume and there are no mutual attractions between the individual molecules and hence no potential energy.

Then energy per unit volume of fluid flow is the sum of the kinetic energy density due to translatory motion of the molecules, $\frac{1}{2}\rho v^2$ and the internal energy density $\rho\varepsilon$.

The equation of motion is reduced to,

$$\rho \frac{D\bar{v}}{Dt} = - \text{grad } p + q\bar{E} + \mu \bar{J} \times \bar{H} \; ; \quad \text{neglecting the gravitational force.}$$

Taking the scalar product of the above equation and the velocity vector $\bar{v}$ we get for the kinetic energy,

$$\rho \frac{D}{Dt} \left(\frac{v^2}{2} \right) = - (\bar{v} . \nabla) \, p + q \, (\bar{E} . \bar{v}) + \mu \left[\bar{v} . \, (\bar{J} \times \bar{H}) \right]$$

The internal energy of the perfect gas depends on its temperature T only.

we know,

$$d\varepsilon = dQ - p \, d\left(\frac{1}{\rho}\right)$$

$$\frac{d\varepsilon}{dt} = \frac{dQ}{dt} - p \frac{d}{dt} \left(\frac{1}{\rho} \right)$$

or,

$$\frac{d\varepsilon}{dt} = \frac{dQ}{dt} + \frac{p}{\rho^2} \frac{d\rho}{dt}$$

The equation of continuity for a compressible fluid is,

$$\frac{d\rho}{dt} + \rho \, \text{div} \, \bar{v} = 0$$

so,

$$\frac{d\varepsilon}{dt} = \frac{dQ}{dt} - \frac{p}{\rho} \nabla . \bar{v}$$

It is an experimental fact that heat flow across an element of fluid surface in δt time is proportional to the gradient in the temperature i.e.

$$\delta Q \propto - \delta S \;\; \delta t \frac{\partial T}{\partial x}$$

or,

$$\delta Q = - k \ \delta A \ \delta t \frac{\partial T}{\partial x}$$

where k is the coefficient of thermal conductivity. The negative sign implies that heat flows from points of higher temperatures to points of lower temperatures.

Consider a closed surface S enclosing an arbitrary volume V. The heat flow out in δt time is,

$$- \delta t \iint k \frac{\partial T}{\partial x} dS = - \delta t \iint k(\nabla T) . dS$$

$$= - \delta t \iiint \nabla .(k \nabla T) \, dV$$

The heat added to the system per unit volume is then,

$$\rho \, \delta Q = \delta t \, \nabla . \left[k \nabla T \right]$$

Therefore, as $\delta t \longrightarrow 0$

$$\rho \frac{dQ}{dt} = \nabla . \left[k \nabla T \right]$$

So, for the internal energy

$$\rho \frac{D\mathcal{E}}{Dt} = \rho \frac{DQ}{Dt} - p \, (\nabla . \bar{v})$$

$$\rho \frac{D\mathcal{E}}{Dt} = \nabla . \left[k \nabla T \right] - p \, (\nabla . \bar{v})$$

Hence the sum of kinetic and internal energy gives,

$$\rho \frac{D}{Dt} \left(\mathcal{E} + \frac{v^2}{2} \right) = - (\bar{v} . \nabla) \, p - p(\nabla . \bar{v}) + \nabla . \left[k \nabla T \right] +$$

$$+ q(\bar{E} . \bar{v}) + \mu \left[\bar{v} . (\bar{J} \times \bar{H}) \right]$$

$$\rho \frac{D}{Dt} \left(\mathcal{E} + \frac{v^2}{2} \right) = - \nabla . (p \bar{v}) + \nabla . \left[k \nabla T \right] + q(\bar{E} . \bar{v}) + \mu \left[\bar{v} . (J \times H) \right] \quad (1.2)$$

Now we find the ordinary energy equation of an electromagnetic field.

When the fluid moves in an electric and magnetic field it experiences a force known as the Lorentz force and experimentally its value is,

$$\overline{F} = q\overline{E} + \frac{1}{c}\,(\,\overline{J} \times \overline{B}\,)$$

Where $q\overline{E}$ is the electric force per unit volume due to an electric field and ($\overline{J} \times \overline{B}$) is the magnetic force per unit volume due to magnetic field.

The rate of work done by the Lorentz force $\overline{F}$ is,

$$\overline{F}.\overline{v} = q\,(\,\overline{E}.\overline{v}\,) + \frac{1}{c}\,[\,\overline{v}.(\overline{J} \times \overline{B})] \qquad (1.3)$$

This force is the cause of the motion of charges if they are free in the field of $\overline{E}$ and $\overline{B}$. If $\overline{v}$ is the velocity of motion of the average charge density q at any point, we have

$$\overline{J} = q\overline{v}$$

so,

$$\overline{F}.\overline{v} = \overline{E}.\,\overline{J} + \frac{\mu q}{c}\,\left[(\,\overline{v}.(\,\overline{v} \times \overline{H}\,)\right]$$

or,

$$\overline{F}.\overline{v} = \overline{E}.\,\overline{J} \quad \text{as } \overline{v}.(\overline{v} \times \overline{H}) = 0 \qquad (1.4)$$

Now we transform the right hand side of above equation (1.4) into a function of $\overline{E}$, $\overline{D}$, $\overline{B}$ and $\overline{H}$ with the help of Maxwell equations.

Since,

$$\text{Curl}\,\overline{H} = \frac{4\pi}{c}\,\overline{J} + \frac{1}{c}\,\frac{\partial \overline{D}}{\partial t}$$

So,

$$\overline{E}.\,\text{Curl}\,\overline{H} = \frac{4\pi}{c}(\overline{E}.\,\overline{J}) + \frac{1}{c}\,(\,\overline{E}.\,\frac{\partial \overline{D}}{\partial t}\,)$$

Using the vector identity, we have

$$\bar{E}.\ \text{Curl}\ \bar{H} = \bar{H}.\ \text{Curl}\ \bar{E} - \text{div}\ (\bar{E} \times \bar{H})$$

with the aid of this result

$$\bar{E}.\bar{J} = \frac{c}{4\pi}\left[\bar{H}.\ \text{Curl}\ \bar{E} - \text{div}\ (\bar{E} \times \bar{H})\right] - \frac{1}{4\pi}\left(\bar{E}.\ \frac{\partial \bar{D}}{\partial t}\right)$$

Since, $\text{Curl}\ \bar{E} = -\dfrac{1}{c}\dfrac{\partial \bar{B}}{\partial t}$

$$\bar{E}.\bar{J} = -\frac{c}{4\pi}\ \text{div}\ (\bar{E} \times \bar{H}) - \frac{1}{4\pi}\left[\left(\bar{H}.\ \frac{\partial \bar{B}}{\partial t}\right) + \left(\bar{E}.\ \frac{\partial \bar{D}}{\partial t}\right)\right]$$

If the medium is isotropic then

$$\frac{\partial \bar{H}}{\partial t} = 0 \quad , \quad \frac{\partial \bar{E}}{\partial t} = 0$$

We write,

$$\bar{E}.\bar{J} = -\frac{c}{4\pi}\ \text{div}\ (\bar{E} \times \bar{H}) - \frac{1}{4\pi}\left[\frac{\partial}{\partial t}(\bar{H}.\bar{B}) + \frac{\partial}{\partial t}(\bar{E}.\bar{D})\right]$$

or,

$$\bar{E}.\bar{J} = -\frac{c}{4\pi}\ \text{div}\ (\bar{E} \times \bar{H}) - \frac{\partial}{\partial t}\left(\frac{\mu H^2 + \epsilon E^2}{8\pi}\right)$$

Since a part of the power density is the rate of variation of $(\epsilon E^2 + \mu H^2)/8\pi$, this quantity must be the sum of energy densities in electric and magnetic fields.

$$\frac{\partial}{\partial t}\left(\frac{\epsilon E^2}{8\pi} + \frac{\mu H^2}{8\pi}\right) = -\left[\frac{c}{4\pi}\text{div}\ (\bar{E} \times \bar{H}) + \bar{E}.\ \bar{J}\right] \qquad (1.5)$$

Using (1.3) and (1.4) equation (1.2) becomes,

$$\rho \frac{D}{Dt}\left(\mathcal{E} + \frac{v^2}{2}\right) = -\ \text{div}\ (\bar{v}\, p) + \bar{F}.\bar{v} + \text{div}\ (k \nabla T)$$

$$= \left[-\ \text{div}\ (\bar{v}\, p) + \bar{E}.\ \bar{J} + \text{div}\ (k \nabla T)\right] \quad (1.6)$$

Adding equations (1.5) and (1.6) we get

$$\rho \frac{D}{Dt} \left(\mathcal{E} + \frac{v^2}{2} \right) + \frac{\partial}{\partial t} \left(\frac{\epsilon E^2}{8\pi} + \frac{\mu H^2}{8\pi} \right) =$$

$$= - \nabla \cdot (\bar{v}\, p) + \nabla \cdot (k \nabla T) - \frac{c}{4\pi} \nabla \cdot (\bar{E} \times \bar{H})$$

This is an equation for total energy per unit volume of the flow field, when there is no heating by radiant energy.

The terms on the left hand side of above equation give the rate of change of energy per unit volume in the field. The right hand side terms of the above equation have the form of divergence so that, on integrating through a certain volume, all of them reduce to surface integrals.

The first term represents the work done by the pressure force, second term represents the heat loss through conduction from the surface and the last term represents the out flow of electromagnetic energy through the surface where $\bar{E} \times \bar{H}$ is the Poynting vector.

The above equation of energy can be written as,

$$\frac{\partial}{\partial t} \left(\rho \mathcal{E} + \frac{1}{2} \rho v^2 \right) + \frac{\partial}{\partial t} \left(\frac{\epsilon E^2}{8\pi} + \frac{\mu H^2}{8\pi} \right) +$$

$$+ \operatorname{div} \left[\rho \bar{v} \left(\mathcal{E} + \frac{1}{2} v^2 \right) + p \bar{v} \right] = \operatorname{div}(k \operatorname{grad} T) - \frac{c}{4\pi} \operatorname{div}(\bar{E} \times \bar{H})$$

For adiabatic flow, no heat is added, conducted or radiated from the flow field. i.e.

$$dQ = 0$$

The energy equation for the adiabatic flow field,

$$\frac{\partial}{\partial t}\left(\rho\varepsilon + \frac{1}{2}\rho v^2 + \frac{\epsilon E^2}{8\pi} + \frac{\mu H^2}{8\pi}\right) + \mathrm{div}\left[\rho\,\bar{\mathbf{v}}\left(\varepsilon + \frac{1}{2}v^2\right) + p\bar{\mathbf{v}}\right] =$$

$$= -\frac{c}{4\pi}\mathrm{div}\,(\bar{\mathbf{E}} \times \bar{\mathbf{H}})$$

In summation convention form,

$$\frac{\partial}{\partial t}\left(\rho\varepsilon + \frac{1}{2}\rho v^2 + \frac{\epsilon E^2}{8\pi} + \frac{\mu H^2}{8\pi}\right) + \frac{\partial}{\partial x_i}\left[\rho v_i\left(\varepsilon + \frac{1}{2}v^2\right) + pv_i\right] =$$

$$= -\frac{c}{4\pi}\frac{\partial}{\partial x_i}\left(\epsilon_{ijk}\, C_i\right)$$

Where, $C_i = E_i \times H_i$

and, $\epsilon_{ijk} = 0 \quad , i \neq j \neq k$

$\quad\quad\;\; = 1 \quad , i = j = k$

3. <u>SHOCK WAVES, THEIR EXISTENCE</u>

If a small disturbance (i.e. with an infinitesimal amplitude and small velocity) is created within a non-viscous isentropic compressible fluid it will propagate throughout the fluid as a wave motion and with the velocity of sound relative to the fluid without suffering any distortion.

In this case we get the wave equations, which shows that both the density and velocity variations follow the same wave patterns, derived from the linearizing of the equations of fluid motion. The solution $\rho(x,t)$ and $u(x,t)$ both are the single valued function. The equations of fluid motion reduced to linear form as velocity is small, so that the term $(\bar{\mathbf{v}}.\mathrm{grad})\bar{\mathbf{v}}$

in Euler's equation of motion may be reglected.

If the assumptions of infinitesimal amplitudes and gradients are removed, then the wave velocity will not be a constant and a simple wave velocity will distort as it propagates.

In this case as velocity is not small, we have to solve the complete non-linear equations of fluid motion. The method of characteristics can be applied to solve such hyperbolic type equations.

A simple analytic solution of these equations of fluid motion may be find such that the density is a function of velocity only. After some manipulation we get equations in u and ρ . General solutions of these equations are,

$$u = f_1 \left[x - (u \pm a)\ t \right]$$

$$\rho = f_2 \left[x - (u \pm a)\ t \right]$$

where f_1 and f_2 are arbitrary functions.

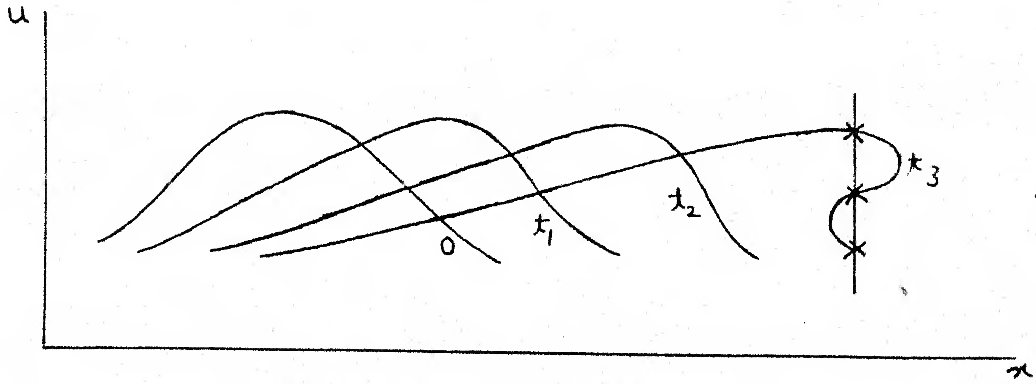
We consider the solution,

$$u = f_1 \left[x - (u + a)\ t \right]$$

This equation shows that the disturbance is propagated at an instantaneous velocity u + a, instantaneous because this velocity is a function of time. If the increment in velocity u is much smaller then speed of sound, then the solution is the case of sound wave i.e. the linear case and the curve $u = f_1\ (x - at)$ does not change its shape as the disturbance propagates. But if u is not small the

shape of the wave is distorted as it propagates.

The resulting distortions in velocity distribution are shown in figure.



FORMATION OF SHOCK

At a time t_1, the points of high velocity move faster to the right than those of lower velocity. Thus the creast has moved faster to the right than the trough and so the profile is disturbed. The compressive part of the wave, where the propagation velocity is a decreasing function of x, distorts to give a triple valued solution for u(x,t), which is physically impossible for longitudinal wave.

Actually, the difficulty is over come by the formation of a ' shock wave '. In such cases, a large variation in pressure and density occurs in a very narrow region in which the flow variables change rapidly and the fluid no longer undergoes isentropic changes. The thickness of this shock wave region is very small and so we may considered the shock wave as a surface of discontinuity for many practical problems of inviscid fluids([4],[7]).

The propagation of shock is faster than sound when observed from one side of the discontinuity and less than that of sound when observed from the other side. Hence the velocity of the shock will be supersonic viewed from

ahead and it will be subsonic viewed from behind.

The formation of these shock waves causes a great noise equal to that of an explosion, usually termed as supersonic bang so the effect of shock must be taken into account in the design of aeroplanes, pipe flow, supersonic flight of projectiles and so on.

For the flow of an inviscid and non-conducting gas the laws of conservation of mass,momentum and energy are originally formulated in the differential equation form as it is assumed that the flow variables defining the flow are continuous functions. Flows are also possible, however, for which discontinuities in the distribution of these flow variables accur. So the conservation laws can also be applied to such discontinuous flow and hence across a shock.

The condition for the existence of shock waves which may be called the jump conditions, relate the velocity, pressure, density and temperature in front of the shock to those behind of them. The jump conditions are the simple consequence of the laws of conservation of mass, momentum and energy across the surface of discontinuity and the equation of state of the medium through which the shock is moving.

4. JUMP CONDITIONS ACROSS A DISCONTINUITY

A differential equation can be represented as

$$\frac{\partial H}{\partial t} + \frac{\partial F}{\partial x} = K \tag{1.7}$$

Where H, F, K are functions of position and time. K contribute for sources or sinks. F is the flux per unit time and H is the distribution of some state of medium. Functions H, F and K are continuous and differentiable.

Let $x_1(t)$ and $x_2(t)$ are continuous and differentiable function of t and $x_1(t) < x_2(t)$ for every t.

Integrating (1.7),

$$\int_{x_1(t)}^{x_2(t)} \frac{\partial H}{\partial t}\, dx + F(x_2) - F(x_1) = \int_{x_1(t)}^{x_2(t)} K \, dx \qquad (1.8)$$

Now we know that Newton-Leibnitz formula is,

$$\frac{\partial}{\partial t}\int_{x_1(t)}^{x_2(t)} H(x, t)\, dx = \int_{x_1(t)}^{x_2(t)} \frac{\partial H}{\partial t}\, dx + H(x_2, t)\frac{dx_2}{dt} - H(x_1, t)\frac{dx_1}{dt}$$

Using this formula, (1.8) becomes

$$\int_{x_1(t)}^{x_2(t)} \frac{\partial H}{\partial t}\, dx + H(x_2, t)\frac{dx_2}{dt} - H(x_1, t)\frac{dx_1}{dt} = F(x_2) - F(x_1) + \int_{x_1(t)}^{x_2(t)} K\, dx$$

If x_1 and x_2 are constants then

$$\frac{\partial}{\partial t}\int_{x_1}^{x_2} H\, dx = F(x_2) - F(x_1) + \int_{x_1}^{x_2} K\, dx \qquad (1.9)$$

Where $F(x_2)$ is the outgoing flux and $F(x_1)$ is the incoming flux.

The above result (1.9) states that,

The time rate of change of total amount of state of the medium in any section $x_1 < x < x_2$ (i.e. L.H.S. $\frac{\partial}{\partial t}\int_{x_1}^{x_2} H\, dx$) is equal to the difference of outgoing flux and incoming flux, and some contribution of sources and sinks.

If there is no sources or sinks i.e. K = 0 and,

$$H = H(U) \quad , \quad F = F(U)$$

then (1.9) represents the conservation of mass.

$$\frac{\partial}{\partial t}\int_{x_1}^{x_2} H\,dx = F(x_2) - F(x_1)$$

and equation (1.7) is in the conservation form,

$$\frac{\partial H}{\partial t} + \frac{\partial F}{\partial x} = 0$$

The most general conservation form for differential equation is,

$$\frac{\partial}{\partial t}(U^n) + \frac{\partial}{\partial x}\left(\frac{n}{n+1}U^{n+1}\right) = 0, \quad n = 1, 2, --- \qquad (1.10)$$

Suppose there is a discontinuity at $x = X(t)$ and x_1 and x_2 are chosen so that $x_1 < X(t) < x_2$. Suppose U^n and U^{n+1} and their first derivatives are continuous in $X(t) > x \geqslant x_1$ and in $x_2 \geqslant x > X(t)$ and have finite limits as $x \longrightarrow X(t)$ from below and above.

Integrating (1.10),

$$\int_{x_1}^{x_2} \frac{\partial}{\partial t} U^n\,dx = \frac{n}{n+1}\left\{U^{n+1}(x_1) - U^{n+1}(x_2)\right\} \qquad (1.11)$$

Also using the property of line integral,

$$\int_{x_1}^{x_2} \frac{\partial U^n}{\partial t}\,dx = \int_{x_1}^{X(t)} \frac{\partial U^n}{\partial t}\,dx + \int_{X(t)}^{x_2} \frac{\partial U^n}{\partial t}\,dx$$

Now we use the Newton-Leibnitz formula on the left hand side

$$\int_{x_1}^{x_2} \frac{\partial U^n}{\partial t}\,dx = \frac{\partial}{\partial t}\int_{x_1}^{X(t)} U^n\,dx + U^n(\bar{X}, t)\frac{dX}{dt} + \frac{\partial}{\partial t}\int_{X(t)}^{x_2} U^n\,dx - U^n(X^+, t)\frac{dX}{dt}$$

Where $U^n(X^-, t)$, $U^n(X^+, t)$ are the value of $U^n(x,t)$ as $x \longrightarrow X(t)$ from left and right, respectively.

$$\int_{x_1}^{x_2} \frac{\partial U^n}{\partial t} dx = \frac{\partial}{\partial t}\int_{x_1}^{X(t)} U^n dx + \frac{\partial}{\partial t}\int_{X(t)}^{x_2} U^n dx + S\,(U_l^n - U_r^n) \quad (1.12)$$

where, $S = \frac{dX}{dt}$

and $\qquad U^n(X^-, t) \longrightarrow U_l^n \;,\; U^n(X^+, t) \longrightarrow U_r^n$

On comparing (1.11) and (1.12) we get,

$$\frac{\partial}{\partial t}\int_{x_1}^{X(t)} U^n dx + \frac{\partial}{\partial t}\int_{X(t)}^{x_2} U^n dx + S\,(U_l^n - U_r^n) = \frac{n}{n+1}\left[U^{n+1}(x_1) - U^{n+1}(x_2)\right]$$

Since $\frac{\partial U^n}{\partial t}$ is bounded in each of the intervals separately, the integral tends to zero in the limit as $x_1 \longrightarrow X^-$ and $x_2 \longrightarrow X^+$. Therefore,

$$S\,(U_l^n - U_r^n) = \frac{n}{n+1}\left[U^{n+1}(x_1) - U^{n+1}(x_2)\right]$$

$$= \frac{n}{n+1}\left[U^{n+1}(X^-) - U^{n+1}(X^+)\right]$$

or

$$S\,(U_l^n - U_r^n) = \frac{n}{n+1}\left(U_l^{n+1} - U_r^{n+1}\right)$$

This condition may also be written as,

$$S\,[U^n] = \frac{n}{n+1}\,[U^{n+1}]$$

or

$$-\,S\,[U^n] + \frac{n}{n+1}\,[U^{n+1}] = 0$$

where the brackets [ ] indicate the jump in quantity and S is the speed of propagation of the discontinuity or shock front.

$$S = \frac{dx}{dt} = \frac{n}{n+1}\left[\frac{U^{n+1}}{U^{n}}\right]$$

This gives the location of the discontinuity. So if the conservation equation is,

$$\frac{\partial H}{\partial t} + \frac{\partial F}{\partial x} = 0 \qquad (1.13)$$

then the jump relation is,[7],

$$-S[H] + [f] = 0 \qquad (1.14)$$

a. JUMP CONDITIONS IN ORDINARY FLUIDS

Consider a stationary shock front (discontinuity) separating two regions, i.e. $S = \frac{dx}{dt} = 0$ so,

the jump condition (1.14) becomes,

$$[f] = 0 \qquad (1.15)$$

The equation representing the conservation of mass, momentum and energy across a surface of discontinuity are given below,

Equation of continuity is,

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i) = 0$$

which is in the form (1.13). So the corresponding jump condition (1.15) is,

$$[\rho v_i n_i] = 0$$

i.e. $\rho_1 v_{1i} n_i = \rho_2 v_{2i} n_i = m$ (1.16) a

Equation of motion is

$$\rho \frac{\partial}{\partial t} v_j + \rho \left(v_i \frac{\partial}{\partial x_i} \right) v_j = - \frac{\partial}{\partial x_i} p$$

or

$$\frac{\partial}{\partial t}(\rho v_j) - v_j \frac{\partial \rho}{\partial t} + \rho v_i \frac{\partial}{\partial x_i} v_j = - \frac{\partial}{\partial x_i} p$$

or

$$\frac{\partial}{\partial t}(\rho v_j) + v_j \frac{\partial}{\partial x_i}(\rho v_i) + \rho v_i \frac{\partial}{\partial x_i} v_j = - \frac{\partial}{\partial x_i} p$$

(using equation of continuity)

or

$$\frac{\partial}{\partial t}(\rho v_j) + \rho v_j \frac{\partial v_i}{\partial x_i} + v_j v_i \frac{\partial \rho}{\partial x_i} + \rho v_i \frac{\partial}{\partial x_i} v_j = - \frac{\partial}{\partial x_i} p$$

or

$$\frac{\partial}{\partial t}(\rho v_j) + \frac{\partial}{\partial x_i}(\rho v_i v_j) = - \frac{\partial}{\partial x_i} p$$

$$\frac{\partial}{\partial t}(\rho v_j) + \frac{\partial}{\partial x_i}(\rho v_i v_j + p) = 0$$

which is in the conservation form, jump condition is

$$\left[(\rho v_i v_j + p) n_i \right] = 0$$

using (1.16) a we have,

$$[p] n_i = - m [v_j] \qquad (1.16)b$$

Equation of energy is,

$$\frac{dh}{dt} = \frac{1}{\rho} \frac{dp}{dt}$$

In conservation form it can be written as

$$\frac{\partial}{\partial t}\left(\rho \varepsilon + \frac{\rho v^2}{2}\right) + \frac{\partial}{\partial x_i}\left\{ \rho v_i \left(\frac{p}{\rho} + \varepsilon + \frac{v^2}{2} \right) \right\} = 0$$

where $\quad h = \varepsilon + \frac{p}{\rho}$

and $\quad \varepsilon = c_v T.$

$$\varepsilon = \frac{c_v p}{\rho R} = \frac{p}{\rho}\left(\frac{c_v}{c_p - c_v}\right)$$

$$= \frac{p}{\rho(\gamma-1)}$$

So,

$$\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{\rho v^2}{2}\right) + \frac{\partial}{\partial x_i}\left\{\rho v_i\left(\frac{\gamma p}{\rho(\gamma-1)} + \frac{v^2}{2}\right)\right\} = 0$$

Jump condition is,

$$\left[\rho v_i\left(\frac{\gamma p}{\rho(\gamma-1)} + \frac{v^2}{2}\right) n_i\right] = 0$$

or

$$\left[\rho v_i n_i\left(\frac{\gamma p}{\rho(\gamma-1)} + \frac{v^2}{2}\right)\right] = 0$$

If $\rho_1 v_{1i} n_i = \rho_2 v_{2i} n_i \neq 0$ then the constant factor may be dropped in the above condition and we have,

$$\left[\frac{\gamma p}{\rho(\gamma-1)} + \frac{v^2}{2}\right] = 0 \qquad (1.16)\,c$$

The equations (1.16) are the jump conditions (Rankine-Hugoniot) when the shock is at rest[8].The subscript 1 and 2 denote a variable ahead and behind of the shock front, n_i are the components of the unit normal to the shock and bracket denotes the difference of values in the two sides of the shock surface of the quantity enclosed.

b. JUMP CONDITIONS IN MAGNETOGASDYNAMICS

In an electrically conducting fluids in the presence of magnetic field,discontinuity (i.e. a shock wave) in flow variables can exists. The study of magnetohydrodynamic shock waves was begun in 1950 with the paper of F. de Hoffmann and Teller [9] . Since then continued interest inspired by astrophysics, by flight at the outer edges of the atmosphere,

etc. has produced many papers describing shock wave properties. The basic properties of magnetogasdynamic shock waves are determined by the conservation laws. In the presence of magnetic field the relations connecting the flow and field quantities on the two sides of the shock surface are as follow [10] ,

Equation of continuity is,

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i) = 0$$

which is in the form (1.13) so the corresponding jump condition (1.15) is,

$$[\rho v_i n_i] = 0$$

or

$$[\rho v_n] = 0$$

i.e. $\rho_1 v_{n_1} = \rho_2 v_{n_2}$ (1.17).a

The equation of motion is,

$$\frac{\partial}{\partial t}(\rho v_j) + v_j \frac{\partial}{\partial x_i}(\rho v_i) + \rho\left(v_i \frac{\partial}{\partial x_i}\right) v_j = -\frac{\partial}{\partial x_i}\left(p + \frac{\mu H^2}{8\pi}\right)\delta_{ij} + $$

$$+ \frac{\mu}{4\pi}\left(H_i \frac{\partial}{\partial x_i}\right) H_j$$

since $\frac{\partial}{\partial x_i}(\rho v_i v_j) = v_j \frac{\partial}{\partial x_i}(\rho v_i) + \rho v_i \frac{\partial}{\partial x_i} v_j$

and $\frac{\partial}{\partial x_i}(H_i H_j) = H_j \frac{\partial}{\partial x_i} H_i + H_i \frac{\partial}{\partial x_i} H_j$

so,

$$H_i \frac{\partial}{\partial x_i} H_j = \frac{\partial}{\partial x_i}(H_i H_j) - H_j \frac{\partial}{\partial x_i} H_i$$

$$= \frac{\partial}{\partial x_i}(H_i H_j)$$

as $\operatorname{div} \bar{H} = 0$

Using these results equation of motion can be written as,

$$\frac{\partial}{\partial t}(\rho v_j) + \frac{\partial}{\partial x_i}(\rho v_i v_j) = -\frac{\partial}{\partial x_i}\left(p + \frac{\mu H^2}{8\pi}\right)\delta_{ij} + $$
$$+ \frac{\mu}{4\pi}\frac{\partial}{\partial x_i}(H_i H_j)$$

or

$$\frac{\partial}{\partial t}(\rho v_j) + \frac{\partial}{\partial x_i}\left(\rho v_i v_j - \frac{\mu}{4\pi} H_i H_j + \left(p + \frac{\mu H^2}{8\pi}\right)\delta_{ij}\right) = 0$$

which is in the conservation form. So the jump condition is,

$$\left[\left(\rho v_i v_j - \frac{\mu}{4\pi} H_i H_j + \left(p + \frac{\mu H^2}{8\pi}\right)\delta_{ij}\right) n_i\right] = 0$$

or

$$\left[\left(p + \frac{\mu H^2}{8\pi}\right)\bar{n} + \rho \bar{v} v_n - \frac{\mu}{4\pi}\bar{H} H_n\right] = 0$$

or

$$[\rho \bar{v} v_n] + \left[\left(p + \frac{\mu H^2}{8\pi}\right)\bar{n}\right] = \frac{\mu}{4\pi}[\bar{H} H_n] \qquad (1.17)\text{b}$$

Also we know

$$\operatorname{div} \bar{H} = 0$$

i.e. $$\frac{\partial}{\partial x_i} H_i = 0$$

Jump condition is,

$$[H_i n_i] = 0$$

or

$$[H_n] = 0 \qquad (1.17)\text{c}$$

Equation for the variation of the magnetic field (when $\sigma \to \infty$) is,

$$\frac{\partial H_j}{\partial t} = H_i \frac{\partial}{\partial x_i} v_j - v_i \frac{\partial}{\partial x_i} H_j - H_j \frac{\partial}{\partial x_i} v_i$$

or

$$\frac{\partial H_j}{\partial t} = \frac{\partial}{\partial x_i}(H_i v_j) - v_j \frac{\partial H_i}{\partial x_i} - \frac{\partial}{\partial x_i}(v_i H_j) + H_j \frac{\partial v_i}{\partial x_i} - H_j \frac{\partial}{\partial x_i} v_i$$

or

$$\frac{\partial H_j}{\partial t} = \frac{\partial}{\partial x_i}\left(H_i v_j - v_i H_j\right)$$

or

$$\frac{\partial H_j}{\partial t} + \frac{\partial}{\partial x_i}\left(v_i H_j - H_i v_j\right) = 0$$

jump condition is,

$$\left[\left(v_i H_j - H_i v_j\right) n_i \right] = 0$$

or

$$\left[\bar{H} v_n - H_n \bar{v} \right] = 0 \qquad (1.17)\text{d}$$

The equation of energy is,

$$\frac{\partial}{\partial t}\left\{ \rho\left(\varepsilon + \tfrac{1}{2}v^2 + \frac{H^2}{8\pi}\right)\right\} + \nabla\cdot\left\{ \bar{v}\left(p + \rho\varepsilon + \tfrac{1}{2}\rho v^2\right) + \frac{c}{4\pi}\bar{E}\times\bar{H}\right\} = 0$$

Ohm's law for a moving medium is,

$$\bar{J} = \sigma\left(\bar{E} + \frac{\mu}{c}\bar{v}\times\bar{H}\right)$$

or

$$\bar{E} = \frac{1}{\sigma}\bar{J} - \frac{\mu}{c}\bar{v}\times\bar{H}$$

$$\therefore \quad \frac{c}{4\pi}\bar{E}\times\bar{H} = \frac{\mu}{4\pi}\bar{H}\times(\bar{v}\times\bar{H}) - \frac{c}{4\pi\sigma}\bar{H}\times\bar{J}$$

also we know

$$\text{curl}\,\bar{H} = \frac{4\pi}{c}\bar{J}$$

(neglecting the displacement current)

$$\frac{c}{4\pi}\bar{E}\times\bar{H} = \frac{\mu}{4\pi}\bar{H}\times(\bar{v}\times\bar{H}) - \frac{c^2}{16\pi^2\sigma}\bar{H}\times(\nabla\times\bar{H})$$

In the case of high conductivity it reduces to

$$\frac{c}{4\pi}\bar{E}\times\bar{H} = \frac{\mu}{4\pi}\bar{H}\times(\bar{v}\times\bar{H})$$

or

$$= \frac{\mu}{4\pi}\left(H^2\bar{v} - (\bar{H}\cdot\bar{v})\bar{H}\right)$$

using this result, energy equation can be written as,

$$\frac{\partial}{\partial t}\left\{ \rho\left(\varepsilon + \tfrac{1}{2}v^2 + \frac{H^2}{8\pi}\right)\right\} + \nabla\cdot\left\{ \bar{v}\left(p + \rho\varepsilon + \tfrac{1}{2}\rho v^2\right) + \frac{\mu}{4\pi}\left(H^2\bar{v} - (\bar{H}\cdot\bar{v})\bar{H}\right)\right\} = 0$$

or

$$\frac{\partial}{\partial t}\left\{\rho\left(\varepsilon+\frac{1}{2}v^2+\frac{H^2}{8\pi}\right)\right\}+\frac{\partial}{\partial x_i}\left\{v_i\left(p+\rho\varepsilon+\frac{1}{2}\rho v^2+\frac{\mu H^2}{4\pi}\right)-\frac{\mu}{4\pi}(H_j\cdot v_j)H_i\right\}=0$$

jump condition is,

$$\left[\left\{v_i\left(p+\rho\varepsilon+\frac{1}{2}\rho v^2+\frac{\mu H^2}{4\pi}\right)-\frac{\mu}{4\pi}(H_j\cdot v_j)H_i\right\}n_i\right]=0$$

or

$$\left[v_n\left(\frac{1}{2}\rho v^2+\rho\varepsilon+\frac{\mu H^2}{4\pi}+p\right)\right]=\frac{\mu}{4\pi}\left[(\bar{v}\cdot\bar{H})H_n\right] \qquad (1.17)\mathrm{e}$$

The equations (1.17) are the shock conditions in the frame of reference in which shock is at rest. The equations (1.17) are also called as the MHD Rankine-Hugoniot relation, which can be written as, from equations (1.17)

$$\frac{p_1}{p_2}=\frac{(\gamma+1)\rho_1-(\gamma-1)\rho_2}{(\gamma+1)\rho_2-(\gamma-1)\rho_1}+\frac{\mu[H]^2(\gamma-1)(\rho_1-\rho_2)}{8\pi p_2\left\{(\gamma+1)\rho_2-(\gamma-1)\rho_1\right\}}$$

or

$$\frac{\rho_1}{\rho_2}=\frac{(\gamma+1)p_1+(\gamma-1)p_2}{(\gamma-1)p_1+(\gamma+1)p_2}-\frac{\mu[H]^2(\gamma-1)(\rho_1-\rho_2)}{8\pi\rho_2\left\{(\gamma-1)p_1+(\gamma+1)p_2\right\}}$$

For $\bar{H}=0$ this relation gives the result for a non-conducting fluid.

The usual situation is that the flow ahead of the shock is known and these conditions are used to determine the flow behind or to determine the flow quantities in terms of one of the flow quantity behind.

5. RADIATION PHENOMENON

At very high temperature, radiation can be considered as a continuous emission of energy in the form of electromagnetic wave which propagate in vacuum with the velocity of light. This energy is called as radiant energy or thermal radiation. Whereas, according to ' quantum theory ' the radiant energy emitted or absorbed is not continuous permitting all possible values, as demanded by the wave theory, but in a discrete quantified form, as integral multiples of an elementary quantum of energy, photon or light quanta. The amount of energy in each quantum being given by the product $h\nu$, where h is Planck's constant and ν the frequency of the radiation.

Thus the quantum theory proposes the particle characteristics of radiation, while the classical theory the wave characteristics, both being required to understand the complex behaviour of radiation. It has also been recognised that radiation reveals itself in different types such as the electrical (radio) waves, infrared, visible, ultraviolet, X-rays and gamma rays. This theory of thermal radiation can be applied to understand the processes which take place in stellar media, to explain the observed luminosity of stars and nuclear explosions and also to high temperature fluid flow.

Radiative transfer and radiative heat exchange have an influence on both the state and the motion of the fluid. This influence is caused by the fact that fluid loses or

gains energy by emitting or absorbing heat. The state of the fluid can be described by the fundamental equations which, in the presence of radiation field, must include the interaction between the radiation and the fluid. There are three different thermal radiation effects on the flow field([11], [12], [13])

Radiant Energy Density E_r :

The radiant energy density per unit mass of the fluid is given by,

$$\frac{E_r}{\rho} = \frac{a\,T^4}{\rho}$$

Where a is Stefan-Boltzman constant.

Radiant Pressure p_r:

According to the well known result of classical electrodynamics the pressure of a radiation field is equal to the one-third of the radiant energy density i.e.

$$p_r = \frac{1}{3} E_r$$

$$= \frac{1}{3} aT^4$$

The radiant energy density and radiant pressure become comparable with the energy density and pressure of the fluid only at extremely high temperature or extremely low gas densities.

Radiation Flux F_r :

The net amount of radiant energy passing through the surface per unit area per unit time is called the radiant

In the energy equation the radiant energy density E_r must be added to the energy density of the fluid and also this equation requires the introduction of a term describing the energy losses by radiation. The energy equation (for adiabatic flow) then becomes,

$$\rho \frac{D}{Dt} \left(\varepsilon + \frac{1}{2} v^2 + \frac{E_r}{\rho} \right) = - \operatorname{div} (p\bar{v}) - q$$

replacing q by the divergence of the flux F_r

$$\rho \frac{D}{Dt} \left(\varepsilon + \frac{1}{2} v^2 + \frac{E_r}{\rho} \right) = - \operatorname{div} (p\bar{v} + \bar{F}_r)$$

6. SPHERICAL AND CYLINDRICAL SHOCK WAVE

Consider the propagation of a shock wave, through a perfect gas, of great intensity (i.e. very strong) resulting from a strong explosion i.e. from the instantaneous release of a large quantity of energy.

When the energy is suddenly released, in an infinitely concentrated form and distribution of density, pressure etc. depends only on the distance from some point then this is the case of spherical shock wave.

When the energy is suddenly released along a line and distribution of all quantities is homogeneous in some direction and has complete axial symmetry about that direction, then this is the case of cylindrical shock.

Since we consider the symmetrical flow (centrally or axially) so there are only two independent variables, namely

r and t. So the one-dimensional fundamental equations governing the adiabatic flow are([5], [14])

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \left(\frac{\partial u}{\partial r} + j \frac{u}{r} \right) = 0 \qquad (1.18)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial \rho}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 \qquad (1.19)$$

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) \frac{p}{\rho^{\gamma}} = 0 \qquad (1.20)$$

or,

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} - a^2 \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0$$

Where j = 0,1, or 2 stands for plane, cylindrical or spherical case, respectively, and r is the radial distance from the centre in spherical case, the radial distance from the line of explosion, in the cylindrical case and x coordinate in plane case; u is radial velocity; ρ , the density.

Similarity principle may be used to reduce these equations to ordinary differential equations.

7. SIMILARITY PRINCIPLE AND SELF-SIMILAR GAS MOTION

It is not always possible to solve non-linear differential equations describing the physics of a motion or a process by using mathematical techniques. An approximate solution of such a problem can be obtained by solving a similar problem which is easier to solve.

We consider the problem of one-dimensional adiabatic flows of a perfect gas with constant specific heat, with

either planer, cylindrical or spherical symmetry. The system of equations for flow of this type is given by (1.18) to (1.20). These gas dynamic equations contain five dimensional quantities p, ρ , u, r and t. The dimensions of three of which are independent; for example density, distance and time, so the equations admit three independent similarity transformation groups of quantities. Dimensional analysis can be used to obtain these groupings. By the successive application of these three groups we can obtain solutions for the different flow similar to each other with altered density, length and time scales. Analogous transformations are made at the same time in the initial and boundary conditions of the problem.

The motion itself may be described by the most general functions of the two variables r and t, $\rho(r,t)$, p (r,t) and u(r,t). These functions also contain the parameters entering the initial and boundary conditions of the problem. They do not depend upon the position r and time t independently but are functions only of the combination (r/t). In other words, the distribution of all quantities with respect to r change with time without changing their form; they remain similar to themselves. This type of motion in which the distribution of the flow variables remain similar to themselves, (i.e. similarity in the motion itself) with time and vary only as a result of changes in scale is called self-similar. Consider the distribution of pressure. The function p (r,t) can be written in the form p(r,t) =

$= \pi(t)\ P\ (r/R)$, where $\pi(t)$ is the scale of the pressure and $R(t)$ is the length scale both depend on time in some manner, and the dimensionless ratio $\frac{p}{\pi} = P\ (r/R)$ is a function of new dimensionless coordinate $\eta = r/R$. Multiplying the variables P and η by the scale functions $\pi(t)$ and $R(t)$, we can obtain from the function $P(\eta)$, independent of time the true pressure distribution. The other flow variables, density and velocity, are expressed similarly [15].

It is a natural question that what requirements must be satisfied by the conditions of a problem in order that the motion be self- similar.

Dimensional analysis is used to answer this question. Since the dimensions of pressure and density contain the unit of mass, at least one of the parameters in the problem must also contain a unit of mass. In many cases this is the constant initial density of the gas ρ_0, which has the dimension $M\ L^{-3}$. Let the parameter containing the unit of mass is a. It can be assumed that its dimensions are $[a] = M\ L^k\ T^s$. The dimensions of the functions p, ρ and u are, $[p] = M\ L^{-1}\ T^{-2}$, $[\rho] = M\ L^{-3}$, and $[u] = LT^{-1}$, we can without any loss of generality, represent them in the form suggested by Sedov [16],

$$p = \frac{a}{r^{k+1}\ t^{s+2}}\ P\ , \quad \rho = \frac{a}{r^{k+3}\ t^{s}}\ G, \quad u = \frac{r}{t}\ V$$

Where P, G and V are dimensionless functions that depend on dimensionless groups containing r, t and the parameters of the problem.

For self similar motion it is possible to reduce a system of partial differential equations to a system of ordinary differential equations for new reduced functions of the similarity variable $\eta = r/R$, $R = R(t)$. The boundary and initial conditions of the problem are made dimensionless and in term transformed into conditions on the new unknown function of η . This simplifies the problem greatly from the mathematical standpoint and in a number of cases makes it possible to find exact analytic solutions.

The problem of a strong explosion represents a typical example of a self-similar motion. This problem was formulated and solved by Sedov [16] and succeeded in finding an exact analytic solution, to the equations of self-similar motion. The same problem was also considered by Stanyukovich in his dissertation [17] and by Taylor [18] , both of whom formulated the equations for the problem and obtained numerical and not analytic solutions.

The parameters in the problem of a strong explosion are the initial density of the gas $\rho_0 \sim ML^{-3}$ and the energy of explosion $E \sim ML^2T^{-2}$. The energy E is always equal to the total energy of the moving gas, and as a result an energy integral appears in the problem. These two parameters can not be combined to yield scaler with dimensions of either length or time. Hence the motion will be self-similar, that is, will be a function of a particular combination of the coordinate r (distance from the center of the explosion) and the time t.

The initial pressure and speed of sound p and c in the problem of a strong explosion are assumed to be equal to zero, and hence these quantities are not parameters of the problem. So the quantity r/t can not serve as the similarity variable. In this case the only dimensional combination which contains only length and time is the ratio of E to ρ_0, with the dimension $[E/\rho_0] = L^5 T^{-2}$. Hence the dimensionless quantity

$$\eta = r\left(\frac{\rho_0}{E t^2}\right)^{1/5}$$

can serve as the similarity variable. The distribution of pressure, density, and gas velocity can be expressed as functions of one dimensionless variable η.

8. CONCEPT OF SELF-GRAVITATION

A fluid can be referred as self-gravitating, when the fluid mass included is large and isolated and the gravitational attraction of other parts of the fluid provides the volume force on any particular fluid element; for example, a gaseous star.

In the case of spherical symmetry the effect of all masses on a particle (point mass) at a distance r from the centre of symmetry is equal to the force of attraction by the point mass placed at the centre of symmetry and having a mass m (r, t).

The behaviour of gravitationally interacting gaseous masses forming a star is given by the appropriate equations of motion([12], [16])

The continuity equation for radial gas motion with spherical symmetry is,

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + 2\frac{u}{r}\right) = 0$$

For the inviscid gas, momentum equation taking gravitational forces into account (according to Newton's law of gravitation gravity g at r is $g = \frac{Gm}{r^2}$) can be written as,

$$\frac{\partial u}{\partial t} + u \frac{\partial \rho}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{Gm}{r^2} = 0$$

Where G is the gravitational constant and m (r,t) is the mass of gas with in the sphere of radius r.

To determine m we use the equation,

$$\frac{\partial m}{\partial r} = 4\pi \rho r^2$$

which expresses the fact that the mass of a spherical shell of radius r and thickness ∂r is $4\pi r^2 \rho \, \partial r$

and the energy equation for adiabatic gas motion,

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^\gamma}\right) + u \frac{\partial}{\partial r}\left(\frac{p}{\rho^\gamma}\right) = 0$$

These are the four equations with four unknowns p, ρ, v and m.

A very large no. of papers have been published by now in which analogous self-similar solution were obtained to explain the adiabatic unsteady flow in self-gravitating gas and analysed for systems of partial differential equations encountered in various problems of Astrophysics such as internal motion in stars, motion of nebulae etc.

CHAPTER-II

A SELF-SIMILAR FLOW OF SELF-GRAVITATING GAS BEHIND A SPHERICAL SHOCK WAVE IN MAGNETOGASDYNAMICS.

1. INTRODUCTION:

The explanation and analysis of the internal motion of stars, the theory of explosion of novae and supernovae stars, the motion of gaseous masses at high relative velocities with shock waves in a gravitational field are the problems which are of the great interest now-a-days. Carrus [19] and Sedov [16] obtained the numerical solution for the self-similar adiabatic flow in a self-gravitating gas. Starting with these results, Ryazanov [20] obtained a particular analytic solution. But this solution does not describe the flow behaviour in general. Singh [21] has discussed the self-similar adiabatic flow of self-gravitating gas in ordinary gas dynamics and has obtained numerical solutions.

In this paper, the propagation of a spherical shock wave, in self-gravitating gas, in a non-uniform atmosphere taking account of magnetic field effects on the physical parameters is considered. It is assumed that magnetic field effected only a portion of sphere enclosing the origin i.e. the point of explosion. Similarity principle has been used to reduce the equations governing the flow (inviscid) to ordinary differential equations and it is assumed that the density varies as the inverse power of distance from the point of explosion.

The total energy of the flow increases with time.

2. EQUATIONS OF THE PROBLEM

The equations governing the adiabatic flow and field are,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho u\right) = 0 \, , \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{H}{\rho}\frac{\partial H}{\partial r} + \frac{H^2}{\rho r} + \frac{Gm}{r^2} = 0 \, , \tag{2.2}$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\frac{\partial u}{\partial r} + \frac{Hu}{r} = 0 \, , \tag{2.3}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^{\gamma}}\right) + u\frac{\partial}{\partial r}\left(\frac{p}{\rho^{\gamma}}\right) = 0 \, , \tag{2.4}$$

$$\frac{\partial m}{\partial r} = 4\pi \rho r^2 . \tag{2.5}$$

The flow variables just ahead of the shock, denoted by suffix zero, i.e. u_0, ρ_0, h_0, m_0 and p_0 are,

$$u_0 = 0 \, , \quad \rho_0 = AR^{-\alpha} \, , \quad h_0 = CR^{-\beta} \tag{2.6}$$

Where R is the shock radius.

From (2.5)

$$\frac{\partial m}{\partial r} = 4\pi \rho r^2$$

at r = R (at shock)

$$\frac{\partial m_0}{\partial R} = 4\pi \rho_0 R^2 = 4\pi A R^{2-\alpha} \qquad \text{as} \quad \rho_0 = AR^{-\alpha}$$

so, ahead the shock,

$$m_0 = \frac{4\pi A R^{3-\alpha}}{(3-\alpha)} \qquad \text{(after integration)} \tag{2.6'}$$

From (2.2)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{H}{\rho}\frac{\partial H}{\partial r} + \frac{H^2}{\rho r} + \frac{Gm}{r^2} = 0$$

at r = R

$$\frac{\partial u_0}{\partial t} + u_0\frac{\partial u_0}{R} + \frac{1}{\rho_0}\frac{\partial p_0}{\partial R} + \frac{H_0}{\rho_0}\frac{\partial H_0}{\partial R} + \frac{H_0^2}{\rho_0 R} + \frac{Gm_0}{R^2} = 0$$

Since, ahead the shock,

$$u_0 = 0 \quad , \quad m_0 = \frac{4\pi A R^{3-\alpha}}{(3-\alpha)} \quad , \quad h_0 = cR^{-\beta} \quad , \quad \rho_0 = AR^{-\alpha}.$$

$$\frac{\partial p_0}{\partial R} = -\left(H_0\frac{\partial H_0}{\partial R} + \frac{H_0^2}{R} + \frac{Gm_0\rho_0}{R^2}\right)$$

$$= -\left(\frac{1}{2}\frac{\partial H_0^2}{\partial R} + \frac{H_0^2}{R} + \frac{Gm_0\rho_0}{R^2}\right)$$

Integrating,

$$p_0 = -\left(\frac{1}{2}H_0^2 + \int\frac{H_0^2}{R}dR + G\int\frac{m_0\rho_0}{R^2}dR\right)$$

$$= -\left(\frac{1}{2}c^2R^{-2\beta} + \int c^2R^{-2\beta-1}dR + \frac{4\pi GA^2}{(3-\alpha)}\int R^{1-2\alpha}dR\right).$$

$$= -\left(\frac{1}{2}c^2R^{-2\beta} + \frac{c^2}{-2\beta}R^{-2\beta} + \frac{4\pi GA^2}{(3-\alpha)(2-2\alpha)}R^{2-2\alpha}\right).$$

$$= -\left[\frac{1}{2}c^2R^{-2\beta}\left(1-\frac{1}{\beta}\right) + \frac{2\pi GA^2}{(3-\alpha)(1-\alpha)}R^{2-2\alpha}\right]$$

$$= -\left[-\frac{1}{2}\frac{c^2}{\beta}R^{-2\beta}(1-\beta) + \frac{2\pi GA^2}{(3-\alpha)(1-\alpha)}R^{2-2\alpha}\right]$$

Or,

$$p_0 = \frac{2\pi G A^2}{(\alpha-1)(3-\alpha)} R^{2-2\alpha} + \frac{c^2}{2\beta}(1-\beta) R^{-2\beta} \qquad (2.6)'''$$

Where $1+\beta = \alpha$

and A, C, α, β are constants

The Rankine-Hugoniot boundary conditions of the shock surface, following Whitham [22] are,

$$u_1 = \frac{\xi-1}{\xi} V \quad , \qquad (2.7)$$

$$\rho_1 = \rho_0 \xi \quad , \qquad (2.8)$$

$$p_1 = \psi \rho_0 V^2 \quad , \qquad (2.9)$$

$$H_1 = H_0 \xi \quad , \qquad (2.10)$$

$$m_1 = m_0 \quad ; \qquad (2.11)$$

Where,

$$\psi = \frac{1}{\gamma M^2} + \frac{2(\xi-1)}{[(\gamma+1)-(\gamma-1)\xi]}\left[\frac{1}{M^2} + \frac{\gamma-1}{4M_A^2}(\xi-1)^2\right] . \qquad (2.12)$$

$V = dR/dt$ is the shock velocity and ξ is given by the quadratic equation,

$$M_A^{-2}(2-\gamma)\xi^2 + \left[(2\gamma-1) + \frac{2}{M^2}\right]\xi - (\gamma+1) = 0 . \qquad (2.13)$$

The Mach number M and Alfven's Mach number M_A are given by,

$$M^2 = \frac{v^2 \rho_0}{\gamma p_0} \quad \text{and} \quad M_A^2 = \frac{v^2 \rho_0}{H_0^2} \tag{2.14}$$

The permeability constant of the medium (μ) is taken to be unity.

3. SIMILARITY TRANSFORMATIONS

For finding out the solutions, some similarity assumptions are made by writing the unknown in the following form,

$$u = \frac{r}{t} U(\eta) \quad , \quad \rho = r^K t^\lambda \Omega(\eta) ,$$
$$m = r^{K+3} t^\lambda W(\eta) \quad , \quad p = r^{K+2} t^{\lambda-2} P(\eta) , \tag{2.15}$$
$$H = r^{(K+2)/2} t^{(\lambda-2)/2} N(\eta) .$$

Where $\quad \eta = r^a t^b . \quad$ (2.16)

and K, λ , a and b are constants and are to be determined from the conditions of the problem.

The total energy E (which increases with time) inside the shock wave of radius R is given by

$$E = 4\pi \int_{r^x}^{R} \left(\frac{1}{2} \rho u^2 + \frac{p}{\gamma-1} + \frac{h^2}{2} - \frac{G m \rho}{r} \right) r^2 dr = Bt^q \; (q \geq 0) \tag{2.17}$$

Where B and q are constants and r^x is the coordinate of inner expanding surface.

In terms of variable η , the total energy E can be expressed as,

$$E = 4\pi \int_{r^\alpha}^{R} \Big(\tfrac{1}{2} r^{K+2} t^{\lambda-2} \Omega U^2 + r^{K+2} t^{\lambda-2} \frac{P}{\gamma-1} + r^{K+2} t^{\lambda-2} N - G r^{2K+2} t^{2\lambda} W\Omega \Big) r^2 \, dr$$

$$= 4\pi \int_{r^\alpha}^{R} \Big[r^{K+4} t^{\lambda-2} \Big(\tfrac{1}{2} U^2 \Omega + \frac{P}{\gamma-1} + N \Big) - G r^{2K+4} t^{2\lambda} W\Omega \Big] r^2 \, dr$$

Since,
$$r = \eta^{1/a} t^{-b/a}$$

$$dr = \frac{1}{a} \eta^{\frac{1}{a}-1} t^{-b/a} d\eta$$

$$dr = \frac{1}{a} \frac{r}{\eta} d\eta.$$

also,
$$r^{K+5} = \eta^{\frac{K+5}{a}} t^{-(K+5)b/a}$$

and
$$r^{2K+5} = \eta^{\frac{2K+5}{a}} t^{-(2K+5)b/a}$$

Substituting these values we get,

$$E = \frac{4\pi}{a} \int_{\eta^\alpha}^{\eta_0} \Big[\frac{r^{K+5} t^{\lambda-2}}{\eta} \Big(\tfrac{1}{2} U^2 \Omega + \frac{P}{\gamma-1} + N \Big) - \frac{G r^{2K+5} t^{2\lambda} W\Omega}{\eta} \Big] d\eta.$$

where η^α and η being the values of η at the expanding surface and shock front, respectively.

$$E = \frac{4\pi}{a} \int_{\eta^\alpha}^{\eta_0} \Big[\eta^{\frac{K+5}{a}-1} t^{\lambda-2-\frac{b}{a}(K+5)} \Big(\tfrac{1}{2} U^2 \Omega + \frac{P}{\gamma-1} + N \Big) - G W \Omega \eta^{\frac{2K+5}{a}-1} t^{2\lambda-(2K+5)\frac{b}{a}} \Big] d\eta.$$

Hence by (2.17),

$$B t^{q} = \frac{4\pi}{a} \int_{\eta^\alpha}^{\eta_0} \Big[\eta^{\frac{K+5}{a}-1} t^{\lambda-2-\frac{b}{a}(K+5)} \Big(\tfrac{1}{2} U^2 \Omega + \frac{P}{\gamma-1} + N \Big) - G W \Omega \eta^{\frac{2K+5}{a}-1} t^{2\lambda-(2K+5)\frac{b}{a}} \Big] d\eta. \quad (2.18)$$

We have,

$$\lambda - 2 - \frac{b}{a}(\kappa+5) = q \times 2 \qquad \text{(A)}$$

and, $$2\lambda - (2\kappa+5)\frac{b}{a} = q \qquad \text{(B)}$$

Subtracting,

$$\left[2\lambda - 4 - (2\kappa+10)\frac{b}{a}\right] - \left[2\lambda - (2\kappa+5)\frac{b}{a}\right] = 2q - q$$

or, $$-4 - 10\frac{b}{a} + 5\frac{b}{a} = q$$

or, $$\frac{a}{b} = -\frac{5}{4+q} \qquad (2.19)$$

We choose η_0 = constant at the shock surface.

also, $\eta = \eta_0$ at $r = R$

$$\eta = r^a t^b$$

so,

$$R = \eta_0^{1/a} t^{-b/a}$$

and, $$\frac{dR}{dt} = \eta_0^{1/a}\left(-\frac{b}{a}\right) t^{-b/a-1}$$

$$\frac{dR}{dt} = -\frac{b}{a}\frac{R}{t}$$

So velocity of shock is,

$$V = -\frac{b}{a}\frac{R}{t} \qquad (2.20)$$

The density distribution (ahead the shock) law is,

$$\rho_0 = A R^{-\alpha}$$

Using (2.15),

$$r^{K}\, t^{\lambda}\, \Omega(\eta) = A R^{-\alpha}$$

or,

$$r^{K}\, t^{\lambda}\, \Omega(\eta_0) = A r^{-\alpha}$$

gives, $K = -\alpha$

and $\lambda = 0$

From equation (2.19), we assume, without any loss of generality

$$a = -5 \qquad \text{and} \qquad b = 4 + q$$

Using (A),

$$\lambda - 2 - \frac{b}{a}(K+5) = q$$

or,

$$0 - 2 + \frac{4+q}{5}(5-\alpha) = q$$

gives,

$$\alpha = \frac{10}{4+q}$$

Hence,

$$R = \eta_0^{1/a}\, t^{-b/a}$$

$$R = \eta_0^{-1/5}\, t^{q+4/5} \qquad (2.21)$$

4. SOLUTIONS OF THE PROBLEM

The equation (2.15) is used to reduce the equations (2.1) to (2.5) into the following forms,

Equation (2.1) is,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho u) = 0$$

Since

$$\rho = r^{K}\, t^{\lambda}\, \Omega(\eta)$$

$$\therefore \quad \frac{\partial p}{\partial t} = \lambda r^{K} t^{\lambda-1} \Omega(\eta) + r^{K} t^{\lambda} \Omega'(\eta) \frac{\partial \eta}{\partial t}$$

$$= \lambda r^{K} t^{\lambda-1} \Omega(\eta) + b\eta r^{K} t^{\lambda-1} \Omega'(\eta)$$

as, $\frac{\partial \eta}{\partial t} = \frac{b}{t}\eta$

and,

$$r^2 \rho u = r^{K+3} t^{\lambda-1} \Omega(\eta)\, U(\eta)$$

$$\therefore \quad \frac{\partial}{\partial r}(r^2 \rho u) = \frac{\partial}{\partial r}\left(r^{K+3} t^{\lambda-1}\right) \Omega(\eta) U(\eta) + r^{K+3} t^{\lambda-1} \frac{\partial}{\partial r}\left(\Omega(\eta) \cdot U(\eta)\right)$$

$$= (K+3)\, r^{K+2} t^{\lambda-1} \Omega(\eta) U(\eta) + r^{K+3} t^{\lambda-1} \left[\Omega'(\eta) \frac{\partial \eta}{\partial r} U(\eta) + \right.$$

$$\left. + U'(\eta) \frac{\partial \eta}{\partial r} \Omega(\eta) \right]$$

Since $\frac{\partial \eta}{\partial r} = \frac{a\eta}{r}$

$$\therefore \quad \frac{\partial}{\partial r}(r^2 \rho u) = (K+3)\, r^{K+2} t^{\lambda-1} \Omega(\eta) U(\eta) + a\eta\, r^{K+2} t^{\lambda-1} \Omega'(\eta)\, U(\eta) +$$

$$+ a\eta\, r^{K+2} U'(\eta) \Omega(\eta) .$$

using these values equation (2.1) reduces to,

$$\lambda r^{K} t^{\lambda-1} \Omega(\eta) + b\eta r^{K} t^{\lambda-1} \Omega'(\eta) + \frac{1}{r^2}\left[(K+3)\, r^{K+2} t^{\lambda-1} \Omega(\eta) U(\eta) + \right.$$

$$+ a\eta\, r^{K+2} t^{\lambda-1} \Omega'(\eta) U(\eta) +$$

$$\left. + a\eta\, r^{K+2} t^{\lambda-1} \Omega(\eta)\, U'(\eta) \right] = 0$$

or,

$$\lambda\,\Omega(\eta) + b\eta\,\Omega'(\eta) + (k+3)\,\Omega(\eta)\,U(\eta) + a\eta\,\Omega'(\eta)\,U(\eta) +$$

$$+ a\eta\,U'(\eta)\,\Omega(\eta) = 0$$

substitute the value of λ , b, k and a we get,

$$\left(\frac{10}{\alpha}\right)\eta\,\Omega' + (3-\alpha)\,\Omega U - 5\eta\,\Omega' U - 5\eta\,U'\Omega = 0$$

or

$$\frac{10\eta}{\alpha}\,\frac{\Omega'}{\Omega} + (3-\alpha)U - 5\eta\,\frac{\Omega'}{\Omega}U - 5\eta\,U' = 0$$

or

$$5\eta\left(\frac{2}{\alpha} - U\right)\frac{\Omega'}{\Omega} + (3-\alpha)U - 5\eta\,U' = 0 \qquad (2.22)$$

Equation (2.2) is,

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{H}{\rho}\frac{\partial H}{\partial r} + \frac{H^2}{\rho r} + \frac{Gm}{r^2} = 0$$

Since $\quad u = \dfrac{r}{t}\,U(\eta)$

$$\therefore \frac{\partial u}{\partial t} = -r t^{-2} U(\eta) + r t^{-1} U'(\eta)\,\frac{\partial \eta}{\partial t}$$

$$= -r t^{-2} U(\eta) + b\eta\, r t^{-2} U'(\eta)$$

$$\text{as } \frac{\partial \eta}{\partial t} = b\eta\, t^{-1}$$

and

$$\frac{\partial u}{\partial r} = \frac{1}{t}U(\eta) + \frac{r}{t}U'(\eta)\,\frac{\partial \eta}{\partial r}$$

$$= t^{-1}U(\eta) + r t^{-1} U'(\eta)\cdot a\eta\, r^{-1}$$

$$\frac{\partial u}{\partial r} = t^{-1}U(\eta) + a\eta\, t^{-1} U'(\eta)$$

Also $$p = r^{K+2}\, t^{\lambda-2}\, P(\eta)$$

$$\therefore \frac{\partial p}{\partial r} = (K+2)\, r^{K+1}\, t^{\lambda-2}\, P(\eta) + r^{K+2}\, t^{\lambda-2}\, P'(\eta)\, \frac{\partial \eta}{\partial r}$$

or
$$\frac{\partial p}{\partial r} = (K+2)\, r^{K+1}\, t^{\lambda-2}\, P(\eta) + a\eta\, r^{K+1}\, t^{\lambda-2}\, P'(\eta)$$

and $$H = r^{K/2+1}\, t^{\lambda/2-1}\, N(\eta)$$

$$\therefore \frac{\partial H}{\partial r} = \left(\frac{K+2}{2}\right) r^{K/2}\, t^{\lambda/2-1}\, N(\eta) + r^{K/2+1}\, t^{\lambda/2-1}\, N'(\eta)\, \frac{\partial \eta}{\partial r}$$

$$= \left(\frac{K+2}{2}\right) r^{K/2}\, t^{\lambda/2-1}\, N(\eta) + a\eta\, r^{K/2}\, t^{\lambda/2-1}\, N'(\eta)$$

Just ahead of the shock we have,

$$p_0 = \frac{2\pi A^2 G}{(\alpha-1)(3-\alpha)}\, R^{2-2\alpha} + \frac{c^2}{2\beta}\,(1-\beta)\, R^{-2\beta}$$

where $$1+\beta = \alpha$$

We assume that motion of the shock wave satisfy the power law

$$\rho_0 = A R^{-\alpha}$$

or
$$A = \rho_0 R^{\alpha}$$

and magnetic field distribution law is

$$h_0 = c R^{-\beta}$$

or
$$c = h_0 R^{\beta}$$

so,

$$p_0 = \frac{2\pi \rho_0^2 R^{2\alpha} G R^{2-2\alpha}}{(\alpha-1)(3-\alpha)} + \frac{h_0^2 R^{2\beta}}{2\beta}\,(1-\beta)\, R^{-2\beta}$$

or

$$\frac{2\pi\rho_0^2 G R^2}{(\alpha-1)(3-\alpha)} = p_0 - \frac{h_0^2(2-\alpha)}{2(\alpha-1)}$$

or
$$4\pi\rho_0^2 G R^2 = (3-\alpha)\left[2p_0(\alpha-1) - h_0^2(2-\alpha)\right]$$

$$\therefore \quad G = \frac{2(3-\alpha)(\alpha-1)}{4\pi\rho_0^2 R^2}\left[p_0 - \frac{h_0^2(2-\alpha)}{2(\alpha-1)}\right]$$

$$G = \frac{2(3-\alpha)(\alpha-1)}{\pi}\left[\frac{p_0}{4R^2\rho_0^2} - \frac{h_0^2(2-\alpha)}{8(\alpha-1)R^2\rho_0^2}\right]$$

or

$$G = \frac{2(3-\alpha)(\alpha-1)b^2}{\pi t^2 a^2}\left[\frac{\gamma p_0 a^2 t^2}{4\gamma\rho_0^2 b^2 R^2} - \frac{(2-\alpha)h_0^2 a^2 t^2}{8(\alpha-1)\rho_0^2 b^2 R^2}\right]$$

$$= \frac{2(3-\alpha)(\alpha-1)b^2}{\pi t^2 a^2}\left[\frac{\gamma p_0}{4\gamma\rho_0\rho_0 v^2} - \frac{(2-\alpha)h_0^2}{8(\alpha-1)\rho_0 v^2\rho_0}\right]$$

as, $v = -\frac{b}{a}\frac{R}{t}$

or

$$G = \frac{2(3-\alpha)(\alpha-1)b^2}{\pi t^2 a^2}\left[\frac{1}{4\gamma\rho_0 M^2} - \frac{(2-\alpha)}{8(\alpha-1)\rho_0 M_A^2}\right]$$

where
$$M^2 = \frac{v^2\rho_0}{\gamma p_0}.$$

and
$$M_A^2 = \frac{v^2\rho_0}{h_0^2}$$

$$G = \frac{2(3-\alpha)(\alpha-1)}{4\pi t^2\rho_0}\frac{b^2}{a^2}\left[\frac{1}{\gamma M^2} - \frac{(2-\alpha)}{2(\alpha-1)M_A^2}\right]$$

or

$$G = \frac{(3-\alpha)(\alpha-1)}{2\pi A}\,\frac{b^2}{a^2}\,\frac{R^{\alpha}}{r^{\alpha}}\,\frac{r^{\alpha}}{t^2}\left[\frac{1}{\gamma M^2} - \frac{(2-\alpha)}{2(\alpha-1)M_A^2}\right]$$

$$G = \frac{(3-\alpha)(\alpha-1)}{2\pi A}\,\frac{b^2}{a^2}\,\frac{1}{\eta^{\alpha}}\,\frac{r^{\alpha}}{t^2}\left[\frac{1}{\gamma M^2} - \frac{(2-\alpha)}{2(\alpha-1)M_A^2}\right]$$

substituting the value of $\frac{\partial u}{\partial t}$, $\frac{\partial u}{\partial r}$, $\frac{\partial p}{\partial r}$, $\frac{\partial H}{\partial r}$, G, u, f and H in equation (2.2) we get,

$$-r t^{-2} U(\eta) + b\eta\, r t^{-2} U'(\eta) + r t^{-1} U(\eta)\left[t^{-1}U(\eta) + a\eta t^{-1} U'(\eta)\right] +$$

$$+ \frac{r^{-K} t^{-\lambda}}{\Omega(\eta)}\left[(K+2) r^{K+1} t^{\lambda-2} P(\eta) + a\eta r^{K+1} t^{\lambda-2} P'(\eta)\right] +$$

$$+ \frac{r^{-K/2+1} t^{-\lambda/2-1} N(\eta)}{\Omega(\eta)}\left[\left(\frac{K+2}{2}\right) r^{K/2} t^{\lambda/2-1} N(\eta) + a\eta r^{K/2} t^{\lambda/2-1} N'(\eta)\right] +$$

$$+ \frac{r^{-K-1} t^{-\lambda}}{\Omega(\eta)}\left[r^{K+2} t^{\lambda-2} N(\eta)\cdot N(\eta)\right] +$$

$$+ \frac{(3-\alpha)(\alpha-1)\, b^2 r^{\alpha-2}}{2\pi A\, t^2 A \eta^{\alpha}}\left[\frac{1}{\gamma M^2} - \frac{(2-\alpha)}{2(\alpha-1)M_A^2}\right]\cdot\left(r^{K+3} t^{\lambda} W(\eta)\right) = 0 .$$

or

$$-r t^{-2} U(\eta) + b\eta\, r t^{-2} U'(\eta) + r t^{-2} U(\eta)\cdot U(\eta) + a\eta\, r t^{-2} U(\eta)\cdot U'(\eta) +$$

$$+ (K+2)\, r t^{-2} \frac{P(\eta)}{\Omega(\eta)} + a\eta \frac{P'(\eta)}{\Omega(\eta)} r t^{-2} + \left(\frac{K+2}{2}\right) r t^{-2} \frac{N(\eta)\cdot N(\eta)}{\Omega(\eta)} +$$

$$+ a\eta\, r t^{-2} \frac{N(\eta) N'(\eta)}{\Omega(\eta)} + r t^{-2} \frac{N(\eta)\cdot N(\eta)}{\Omega(\eta)} +$$

$$+ \frac{(3-\alpha)(\alpha-1)}{2\pi A}\,\frac{b^2}{a^2}\,\frac{1}{\eta^{\alpha}}\, r^{K+\alpha+1} t^{\lambda-2}\left[\frac{1}{\gamma M^2} - \frac{(2-\alpha)}{2(\alpha-1)M_A^2}\right] W(\eta) = 0 .$$

or

$$-U + by U' + U^2 + ay UU' + (K+2)\frac{P}{\Omega} + ay\frac{P'}{\Omega} +$$

$$+ \left(\frac{K+2}{2}\right)\frac{N^2}{\Omega} + ay\frac{NN'}{\Omega} + \frac{N^2}{\Omega} +$$

$$+ \frac{(3-\alpha)(\alpha-1)}{2\pi A}\frac{b^2}{a^2}\frac{1}{y^\alpha} r^{\alpha+K} t^\lambda \left[\frac{1}{\gamma M^2} - \frac{(2-\alpha)}{2(\alpha-1)M_A^2}\right] W = 0$$

substituting the value of a, b, k, and λ

$$-U + \left(\frac{10}{\alpha}\right) y U' + U^2 - 5y UU' + (2-\alpha)\frac{P}{\Omega} - 5y\frac{P'}{\Omega} +$$

$$+ \left(1 - \frac{\alpha}{2}\right)\frac{N^2}{\Omega} - 5y\frac{NN'}{\Omega} + \frac{N^2}{\Omega} +$$

$$+ \frac{(3-\alpha)(\alpha-1)}{2\pi A}\frac{100}{25\alpha^2}\frac{1}{y^\alpha}\left[\frac{1}{\gamma M^2} - \frac{(2-\alpha)}{2(\alpha-1)M_A^2}\right] W = 0$$

or

$$5yU'\left(\frac{2}{\alpha} - U\right) + U(U-1) + (2-\alpha)\frac{P}{\Omega} - 5y\frac{P'}{\Omega} -$$

$$- \frac{N^2}{\Omega}\left(\frac{\alpha-4}{2} + 5y\frac{N'}{N}\right) + \frac{2(3-\alpha)(\alpha-1)}{\pi\alpha^2}\frac{1}{y^\alpha} L\frac{W}{A} = 0 \qquad (2.23)$$

where $L = \left[\frac{1}{\gamma M^2} - \frac{(2-\alpha)}{2(\alpha-1)M_A^2}\right]$

Equation (2.3) is,

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\frac{\partial u}{\partial r} + \frac{Hu}{r} = 0$$

Since,

$$H = r^{\frac{K+2}{2}} t^{\frac{\lambda-2}{2}} N(y)$$

$$\therefore \frac{\partial H}{\partial t} = \left(\frac{\lambda-2}{2}\right) r^{\frac{K+2}{2}} t^{\lambda/2-2} N(y) + r^{\frac{K+2}{2}} t^{\frac{\lambda-2}{2}} N'(y)\frac{\partial y}{\partial t}$$

$$\frac{\partial H}{\partial t} = \left(\frac{\lambda-2}{2}\right) r^{K/2+1} t^{\lambda/2-2} N(y) + by\, r^{K/2+1} t^{\lambda/2-2} N'(y)$$

also,

$$\frac{\partial H}{\partial r} = \left(\frac{K+2}{2}\right) r^{K/2} \, t^{\lambda/2 - 1} N(y) + a y \, t^{\frac{\lambda}{2}-1} r^{K/2} N'(y)$$

and

$$\frac{\partial u}{\partial r} = t^{-1} U(y) + a y \, t^{-1} U'(y)$$

using these relations equation (2.3) can be written as,

$$\left(\frac{\lambda-2}{2}\right) r^{\frac{K}{2}+1} t^{\frac{\lambda}{2}-2} N(y) + b y \, r^{\frac{K}{2}+1} t^{\frac{\lambda}{2}-2} N'(y) +$$

$$+ \left(\frac{K+2}{2}\right) r^{K/2+1} t^{\lambda/2-2} N(y) U(y) + a y \, r^{\frac{K}{2}+1} t^{\frac{\lambda}{2}-2} N'(y) U(y) +$$

$$+ r^{\frac{K}{2}+1} t^{\frac{\lambda}{2}-2} N(y) U(y) + a y \, r^{\frac{K}{2}+1} t^{\frac{\lambda}{2}-2} U'(y) N(y) +$$

$$+ r^{\frac{K}{2}+1} t^{\frac{\lambda}{2}-2} N(y) U(y) = 0$$

or

$$\left(\frac{\lambda-2}{2}\right) N(y) + b y N'(y) + \left(\frac{K+2}{2}\right) N(y) U(y) + a y N'(y) U(y) +$$

$$+ 2 N(y) U(y) + a y U'(y) N(y) = 0$$

substituting the value of a, b, λ and k we get,

$$-N + \frac{10}{\alpha} y N' + \left(1 - \frac{\alpha}{2}\right) N U - 5 y N' U + 2 N U - 5 y U' N = 0$$

or

$$-1 + \frac{10}{\alpha} y \frac{N'}{N} + \left(1 - \frac{\alpha}{2}\right) U + 2U - 5 y U \frac{N'}{N} - 5 y U' = 0$$

or

$$5 y \frac{N'}{N}\left(\frac{2}{\alpha} - U\right) + 2U - 5 y U' + (U - 1) - \frac{\alpha}{2} U = 0 \qquad (2.24)$$

Equation (2.4) is,

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} = \frac{\gamma p}{\rho}\left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r}\right).$$

Now, $$\frac{\partial p}{\partial t} = \frac{\partial}{\partial t}\left(r^{K+2}\, t^{\lambda-2}\, P(y)\right)$$

$$= r^{K+2}\left[(\lambda-2)\, t^{\lambda-3} P(y) + t^{\lambda-2} P'(y)\, \frac{by}{t}\right]$$

$$\frac{\partial p}{\partial t} = r^{K+2}\, t^{\lambda-3}\left[(\lambda-2)\, P(y) + by\, P'(y)\right]$$

also we have calculated

$$\frac{\partial p}{\partial r} = r^{K+1}\, t^{\lambda-2}\left[(K+2)\, P(y) + ay\, P'(y)\right]$$

$$\therefore u\, \frac{\partial p}{\partial r} = \frac{r}{t}\, r^{K+1}\, t^{\lambda-2}\, U(y)\left[(K+2)\, P(y) + ay\, P'(y)\right]$$

$$= r^{K+2}\, t^{\lambda-3}\left\{U(y)\left[(K+2)\, P(y) + ay\, P'(y)\right]\right\}$$

and

$$\frac{\gamma p}{\rho} = \gamma\, \frac{r^{K+2}\, t^{\lambda-2}\, P(y)}{r^{K}\, t^{\lambda}\, \Omega(y)}$$

$$= \gamma\, \frac{P(y)}{\Omega(y)}\, \frac{r^2}{t^2}$$

also

$$\frac{\partial \rho}{\partial t} = r^{K}\, t^{\lambda-1}\left[\lambda\, \Omega(y) + by\, \Omega'(y)\right]$$

and

$$\frac{\partial \rho}{\partial r} = \frac{\partial}{\partial r}\left(r^{K}\, t^{\lambda}\, \Omega(y)\right)$$

$$= t^{\lambda}\left[K\, r^{K-1}\, \Omega(y) + r^{K}\, \Omega'(y)\, \frac{ay}{r}\right]$$

$$\frac{\partial \rho}{\partial r} = r^{K-1}\, t^{\lambda}\left[K\, \Omega(y) + ay\, \Omega'(y)\right]$$

$$\therefore u\, \frac{\partial \rho}{\partial r} = r^{K}\, t^{\lambda-1}\, U(y)\left[K\, \Omega(y) + ay\, \Omega'(y)\right]$$

substituting these values in (2.4)

$$r^{K+2}\, t^{\lambda-3}\left[(\lambda-2)P(y)+by\,P'(y)\right]+r^{K+2}t^{\lambda-3}\left\{U(y)\left[(K+2)P(y)+ay\,P'(y)\right]\right\}=$$

$$=\gamma\,\frac{P(y)}{\Omega(y)}\,\frac{r^2}{t^2}\,r^K t^{\lambda-1}\left[\lambda\,\Omega(y)+by\,\Omega'(y)+K\,U(y)\,\Omega(y)+ay\,\Omega'(y)U(y)\right]$$

or

$$(\lambda-2)\,P(y)+by\,P'(y)+(K+2)P(y)U(y)+ay\,P'(y)U(y)=\gamma\lambda P(y)+$$

$$+\gamma by\,P(y)\,\frac{\Omega'(y)}{\Omega(y)}+\gamma K\,P(y)U(y)+\gamma ay\,PU\frac{\Omega'}{\Omega}$$

or

$$(\lambda-2)+by\,\frac{P'}{P}+(K+2)U+ayU\frac{P'}{P}=\gamma\lambda+\gamma by\,\frac{\Omega'}{\Omega}+\gamma KU+\gamma ayU\frac{\Omega'}{\Omega}$$

or

$$(\lambda-2)+(K+2)U+(b+aU)y\,\frac{P'}{P}=\gamma\left[(\lambda+KU)+(aU+b)y\,\frac{\Omega'}{\Omega}\right]$$

substitute the value of constants λ , k, a, and b we have,

$$-2+(2-\alpha)U+\left(-5U+\frac{10}{\alpha}\right)y\,\frac{P'}{P}-\gamma\left[-U\alpha+\left(-5U+\frac{10}{\alpha}\right)y\,\frac{\Omega'}{\Omega}\right]=0$$

or

$$5y\,\frac{P'}{P}\left(\frac{2}{\alpha}-U\right)+U(2-\alpha)-2-\gamma\left[-U\alpha+5y\,\frac{\Omega'}{\Omega}\left(\frac{2}{\alpha}-U\right)\right]=0 \qquad (2.25)$$

Equation (2.5) is,

$$\frac{\partial m}{\partial r}=4\pi\rho r^2$$

$$m=r^{K+3}\,t^{\lambda}\,W(y)$$

$$\frac{\partial m}{\partial r}=(K+3)\,r^{K+2}t^{\lambda}\,W(y)+r^{K+3}t^{\lambda}\,W'(y)\,\frac{\partial y}{\partial r}$$

$$\frac{\partial m}{\partial r}=(K+3)\,r^{K+2}t^{\lambda}\,W(y)+ay\,r^{K+2}t^{\lambda}\,W'(y)$$

substituting the value of $\frac{\partial m}{\partial r}, \rho$ in equation (2.5),

$$(K+3)\, r^{K+2} t^{\lambda} W(\eta) + a\eta\, r^{K+2} t^{\lambda} W'(\eta) = 4\pi\, r^{K+2} t^{\lambda} \Omega(\eta)$$

or

$$(K+3)\, W(\eta) + a\eta\, W'(\eta) = 4\pi\, \Omega(\eta)$$

or

$$(K+3) + a\eta \frac{W'}{W} = 4\pi \frac{\Omega}{W}$$

put $K = -\alpha$ and $a = -5$ we get,

$$5\eta \frac{W'}{W} - (3-\alpha) + 4\pi \frac{\Omega}{W} = 0 \tag{2.26}$$

The shock conditions (2.7) to (2.11) are transformed, at $\eta = \eta_0$

$$u_1 = \frac{\xi - 1}{\xi} V \qquad \text{(by 2.7)}$$

$$\frac{r}{t} U(\eta) = \frac{\xi-1}{\xi} \times -\frac{b}{a} \frac{R}{t}$$

when $\eta = \eta_0$, $R = r$

$$\frac{r}{t} U(\eta_0) = -\frac{\xi-1}{\xi} \times \frac{b}{a} \frac{r}{t}$$

$$U(\eta_0) = -\frac{10}{\alpha} \times -\frac{1}{5} \cdot \frac{\xi-1}{\xi}$$

or

$$U(\eta_0) = \frac{\xi-1}{\xi} \cdot \frac{2}{\alpha} \tag{2.27}$$

By (2.8),

$$\rho_1 = \rho_0 \xi$$

$$r^{K} t^{\lambda} \Omega(\eta) = A R^{-\alpha} \xi$$

when $\eta = \eta_0$ $R = r$

$$r^{\kappa}\, t^{\lambda}\, \Omega(\eta_0) = A\, r^{-\alpha}\, \xi$$

substitute $\kappa = -\alpha$, $\lambda = 0$

$$r^{-\alpha}\, \Omega(\eta_0) = A\, r^{-\alpha}\, \xi$$

or

$$\frac{\Omega(\eta_0)}{A} = \xi \qquad (2.28)$$

By shock condition (2.9)

$$p_1 = \psi \rho_0 V^2$$

$$r^{\kappa+2}\, t^{\lambda-2}\, P(\eta) = \psi A R^{-\alpha} \cdot \frac{b^2}{a^2} \cdot \frac{r^2}{t^2}$$

$$r^{\kappa+2}\, t^{\lambda-2}\, P(\eta_0) = \psi A\, r^{-\alpha+2}\, t^{-2} \cdot \frac{b^2}{a^2}$$

put, $\lambda = 0$, $a = -5$, $b = \frac{10}{\alpha}$ and $k = -\alpha$

$$r^{-\alpha+2}\, t^{-2}\, P(\eta_0) = \psi A\, r^{-\alpha+2}\, t^{-2} \cdot \frac{100}{25\alpha^2}$$

or

$$\frac{P(\eta_0)}{A} = \psi \cdot \frac{4}{\alpha^2} \qquad (2.29)$$

By shock condition,

$$H_1 = H_0\, \xi$$

Since $M_A^2 = \dfrac{V^2 \rho_0}{H_0^2}$

$$H_0 = \frac{V\sqrt{\rho_0}}{M_A}$$

$$H_0 = \frac{-b/a \cdot r/t \cdot R^{-\alpha/2} \cdot A^{1/2}}{M_A}$$

$$H_1 = -\frac{b}{a}\, \frac{r}{t}\, R^{-\alpha/2}\, A^{1/2}\, \frac{1}{M_A} \cdot \xi$$

$$r^{\frac{\kappa+2}{2}}\, t^{\frac{\lambda-2}{2}}\, N(\eta_0) = -\frac{b}{a}\, r^{-\alpha/2+1}\, t^{-1}\, A^{1/2}\, \frac{1}{M_A} \cdot \xi$$

substituting $K = -\alpha$, $\lambda = 0$, $b = \frac{10}{\alpha}$, $a = -5$

$$r^{-\frac{\alpha}{2}+1} t^{-1} N(\eta_0) = \frac{10}{5\alpha} r^{-\frac{\alpha}{2}+1} t^{-1} A^{1/2} \cdot \frac{\xi}{M_A}$$

or

$$\frac{N(\eta_0)}{A^{1/2}} = \frac{2\xi}{\alpha} M_A^{-1} \qquad (2.30)$$

By shock condition (2.11)

$$m_1 = m_0$$

$$r^{K+3} t^{\lambda} W(\eta) = \frac{4\pi A R^{3-\alpha}}{(3-\alpha)}$$

$$r^{K+3} t^{\lambda} W(\eta_0) = \frac{4\pi A r^{3-\alpha}}{(3-\alpha)}$$

$$r^{-\alpha+3} W(\eta_0) = \frac{4\pi A r^{3-\alpha}}{(3-\alpha)} \quad \text{as } \lambda = 0$$

or,

$$\frac{W(\eta_0)}{A} = \frac{4\pi}{(3-\alpha)} \qquad (2.31)$$

In the form of non-dimensional equation (2.22) to (2.26) are to be transformed.

By (2.22)

$$5\eta \frac{\Omega'/A}{\Omega/A}\left(\frac{2}{\alpha} - U\right) + (3-\alpha) U - 5\eta U' = 0$$

or

$$5\eta \frac{\bar{\Omega}'}{\bar{\Omega}}\left(\frac{2}{\alpha} - U\right) + (3-\alpha)U - 5\eta U' = 0 \qquad (2.32)$$

By (2.23) we get

$$5\eta U'\left(\frac{2}{\alpha} - U\right) + U(U-1) + (2-\alpha)\frac{P/A}{\Omega/A} - 5\eta \frac{P'/A}{\Omega/A} -$$

$$- \frac{(N/\sqrt{A})^2}{\Omega/A}\left(\frac{\alpha - U}{2} + 5\eta \frac{N'/\sqrt{A}}{N/\sqrt{A}}\right) + \frac{2(3-\alpha)(\alpha-1)}{\pi\alpha^2} \frac{1}{\eta^{\alpha}} L \frac{W}{A} = 0$$

or

$$5y\,U'\left(\frac{2}{\alpha}-U\right)+U(U-1)+(2-\alpha)\frac{\bar{P}}{\Omega}-5y\,\frac{\bar{P}'}{\Omega}-$$

$$-\frac{\bar{N}^2}{\Omega}\left(\frac{\alpha-4}{2}+5y\,\frac{\bar{N}'}{\bar{N}}\right)+\frac{2(3-\alpha)(\alpha-1)}{\pi\alpha^2}\frac{1}{y^\alpha}L\,\bar{W}=0 \quad (2.33)$$

By (2.24)

$$5y\,\frac{N'/\sqrt{A}}{N/\sqrt{A}}\left(\frac{2}{\alpha}-U\right)+2U-5yU'+(U-1)-\frac{\alpha}{2}U=0$$

or

$$5y\,\frac{\bar{N}'}{\bar{N}}\left(\frac{2}{\alpha}-U\right)+2U-5yU'+(U-1)-\frac{\alpha}{2}U=0 \quad (2.34)$$

By (2.25)

$$5y\,\frac{P'/A}{P/A}\left(\frac{2}{\alpha}-U\right)+U(2-\alpha)-2-\gamma\left[-\alpha U+5y\,\frac{\Omega'/A}{\Omega/A}\left(\frac{2}{\alpha}-U\right)\right]=0$$

or

$$5y\,\frac{\bar{P}'}{\bar{P}}\left(\frac{2}{\alpha}-U\right)+U(2-\alpha)-2-\gamma\left[-\alpha U+5y\,\frac{\bar{\Omega}'}{\bar{\Omega}}\left(\frac{2}{\alpha}-U\right)\right]=0 \quad (2.35)$$

By (2.26) we get,

$$5y\,\frac{W'/A}{W/A}-(3-\alpha)+4\pi\,\frac{\Omega/A}{W/A}=0$$

or

$$5y\,\frac{\bar{W}'}{\bar{W}}-(3-\alpha)+4\pi\,\frac{\bar{\Omega}}{\bar{W}}=0 \quad (2.36)$$

where

$$L=\frac{1}{\gamma M^2}-\frac{(2-\alpha)}{2(\alpha-1)}\frac{1}{M_A^2} \quad (2.37)$$

$$\bar{\Omega}' = \Omega'/A \quad , \quad \bar{P}' = P'/A \quad , \quad \bar{W}' = W'/A \quad , \quad \bar{N}' = N'/\sqrt{A}$$

$$\bar{\Omega} = \Omega/A \quad , \quad \bar{P} = P/A \quad , \quad \bar{W} = W/A \quad , \quad \bar{N} = N/\sqrt{A} .$$

and prime denotes the differentiation with respect to η .

The jump conditions at the shock are,

$$U(\eta_0) = \frac{2}{\alpha}\left(\frac{\xi-1}{\xi}\right) \quad , \quad \bar{\Omega}(\eta_0) = \xi \quad , \quad \bar{P}(\eta_0) = \frac{4}{\alpha^2}\psi \, ,$$

(2.38)

$$\bar{N}(\eta_0) = \frac{2\xi}{\alpha} M_A^{-1} \quad , \quad \bar{W}(\eta_0) = \frac{4\pi}{(3-\alpha)} \, ;$$

Equations of motion are,

$$5\eta \frac{\bar{\Omega}'}{\bar{\Omega}}\left(\frac{2}{\alpha} - U\right) + (3-\alpha)U - 5\eta U' = 0 \, ,$$

$$5U'\eta\left(\frac{2}{\alpha} - U\right) + U(U-1) + \frac{\bar{P}}{\bar{\Omega}}(2-\alpha) - 5\eta\frac{\bar{P}'}{\bar{\Omega}} -$$

$$- \frac{\bar{N}^2}{\bar{\Omega}}\left(\frac{\alpha-4}{2} + 5\eta\frac{\bar{N}'}{\bar{N}}\right) + \frac{2(3-\alpha)(\alpha-1)}{\pi\alpha^2}\frac{1}{\eta^{\alpha}} L\bar{W} = 0 ,$$

$$5\eta \frac{\bar{N}'}{\bar{N}}\left(\frac{2}{\alpha} - U\right) + 2U - 5\eta U' + (U-1) - \frac{1}{2}\alpha U = 0 \, ,$$

$$5\eta \frac{\bar{P}'}{\bar{P}}\left(\frac{2}{\alpha} - U\right) + U(2-\alpha) - 2 - \gamma\left[-\alpha U + 5\eta\frac{\bar{\Omega}'}{\bar{\Omega}}\left(\frac{2}{\alpha} - U\right)\right] = 0 ,$$

and $$5\eta\frac{\bar{W}'}{\bar{W}} + 4\pi\frac{\bar{\Omega}}{\bar{W}} - (3-\alpha) = 0 \, ;$$

5. NUMERICAL RESULTS FOR THE SIMILARITY SOLUTIONS OF THE PROBLEM

For numerical calculation the flow and field variables have been used in the following non-dimensional forms,

$$\frac{u}{u_1} = \frac{(r/t)\, U(\eta)}{(R/t)\, U(\eta_0)}$$

$$\frac{u}{u_1} = \frac{r}{R} \; \frac{U(\eta)/A}{U(\eta_0)/A}$$

since $\quad \eta = r^a t^b \quad$ and $\quad \eta_0 = R^a t^b$

$$\therefore \; \frac{\eta}{\eta_0} = \left(\frac{r}{R}\right)^a$$

so

$$\frac{r}{R} = \left(\frac{\eta}{\eta_0}\right)^{1/a}$$

$$\therefore \; \frac{u}{u_1} = \left(\frac{\eta}{\eta_0}\right)^{1/a} \frac{\bar{U}(\eta)}{\bar{U}(\eta_0)} ,$$

$$\frac{\rho}{\rho_1} = \left(\frac{r}{R}\right)^{K} \frac{\Omega(\eta)}{\Omega(\eta_0)}$$

$$= \left(\frac{\eta}{\eta_0}\right)^{K/a} \cdot \frac{\Omega(\eta)/A}{\Omega(\eta_0)/A}$$

as $\quad K = -\alpha \quad$ we have

$$\frac{\rho}{\rho_1} = \left(\frac{\eta}{\eta_0}\right)^{-\alpha/a} \frac{\bar{\Omega}(\eta)}{\bar{\Omega}(\eta_0)} ,$$

$$\frac{p}{p_1} = \frac{r^{K+2}\, t^{\lambda-2}\, P(\eta)}{R^{K+2}\, t^{\lambda-2}\, P(\eta_0)}$$

or

$$\frac{p}{p_1} = \left(\frac{r}{R}\right)^{K+2} \frac{P(\eta)/A}{P(\eta_0)/A}$$

$$= \left(\frac{\eta}{\eta_0}\right)^{\frac{K+2}{a}} \frac{\overline{P}(\eta)}{\overline{P}(\eta_0)}$$

$$\frac{p}{p_1} = \left(\frac{\eta}{\eta_0}\right)^{\frac{2-\alpha}{a}} \frac{\overline{P}(\eta)}{\overline{P}(\eta_0)} ,$$

$$\frac{H}{H_1} = \frac{r^{K+2/2}\, t^{\lambda-2/2}\, N(\eta)}{R^{K+2/2}\, t^{\lambda-2/2}\, N(\eta_0)}$$

or

$$= \left(\frac{r}{R}\right)^{\frac{K+2}{2}} \frac{N(\eta)/A^{1/2}}{N(\eta_0)/A^{1/2}}$$

$$\frac{H}{H_1} = \left(\frac{\eta}{\eta_0}\right)^{\frac{2-\alpha}{2a}} \frac{\overline{N}(\eta)}{\overline{N}(\eta_0)} ,$$

Similarly,

$$\frac{m}{m_1} = \left(\frac{r}{R}\right)^{K+3} \frac{W(\eta)}{W(\eta_0)}$$

$$= \left(\frac{\eta}{\eta_0}\right)^{\frac{3-\alpha}{a}} \frac{W(\eta)/A}{W(\eta_0)/A}$$

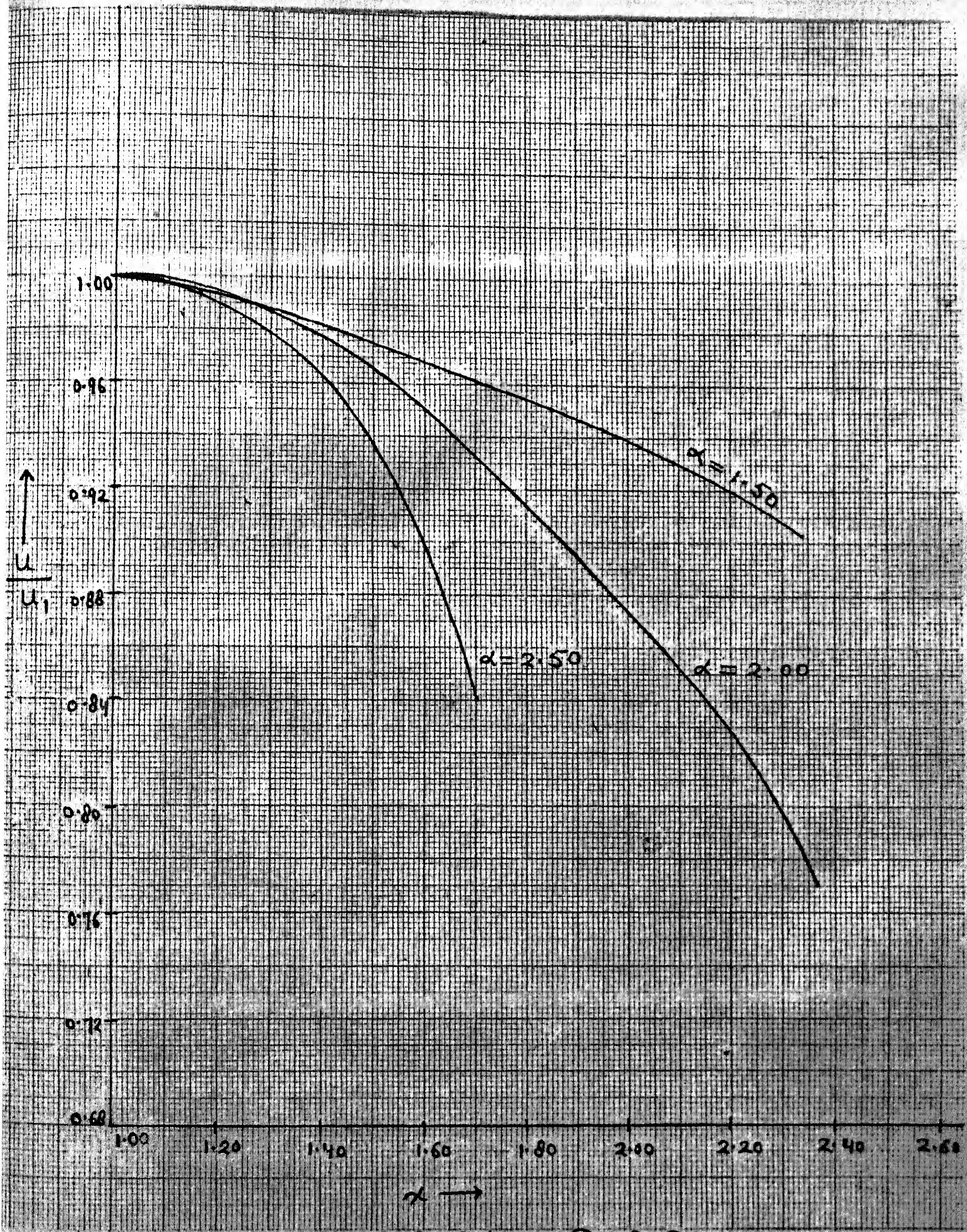
$$\frac{m}{m_1} = \left(\frac{\eta}{\eta_0}\right)^{\frac{3-\alpha}{a}} \frac{\overline{W}(\eta)}{\overline{W}(\eta_0)} ;$$

Numerical integration was performed on a computer, using the Runge-Kutta programme for the three cases $\alpha = 1.5$, $\alpha = 2$ and $\alpha = 2.5$ also for the simplicity it was assumed that $\eta = \eta_0 = 1$. The other constants are $M^2 = 20$, $M_A^{-2} = 0.01$,

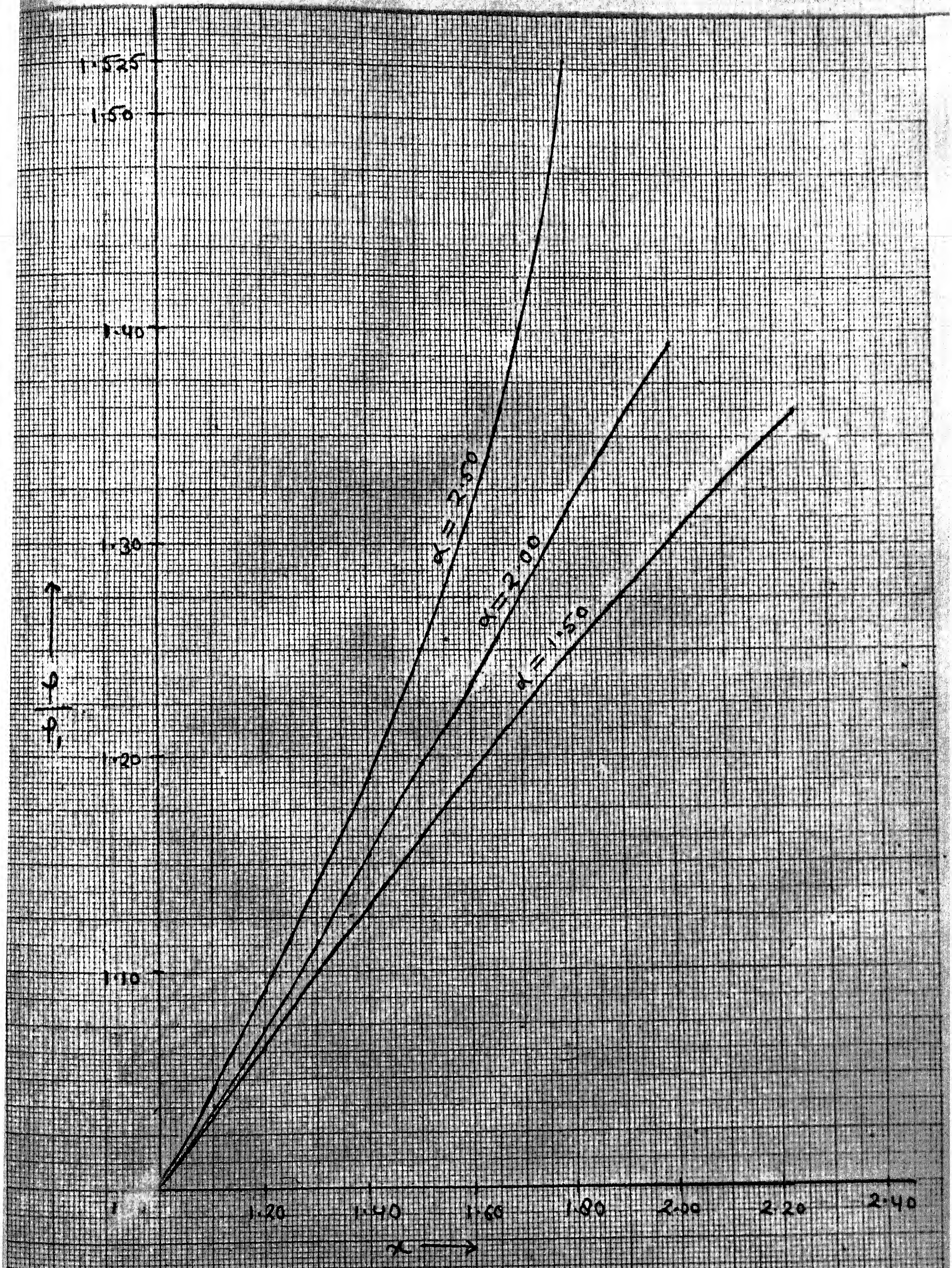
$\gamma = 5/3$ and $a = -5$. The graphs variations of velocity, pressure and density, temperature, magnetic field and mass are shown in the figures No. (1) - (5).

6. CONCLUSION

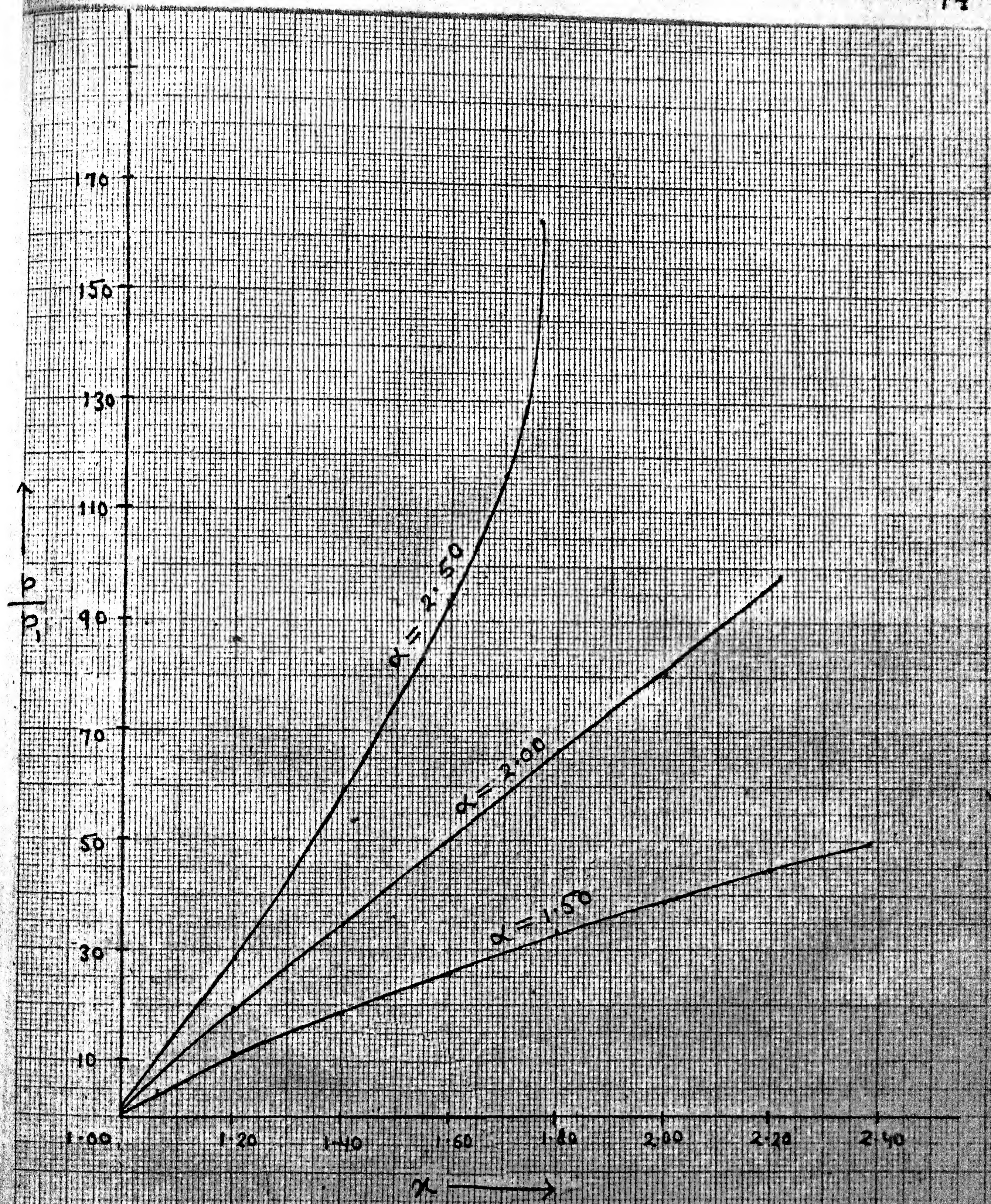
In the problem of propagation of a spherical magnetogas-dynamic shock wave, the velocity and mass are maximum at the shock surface and decrease on moving in opposite direction to the point of explosion from the shock surface whereas the density, pressure and magnetic field are minimum at the shock surface and increase on moving in opposite direction to the point of explosion from the shock surface.



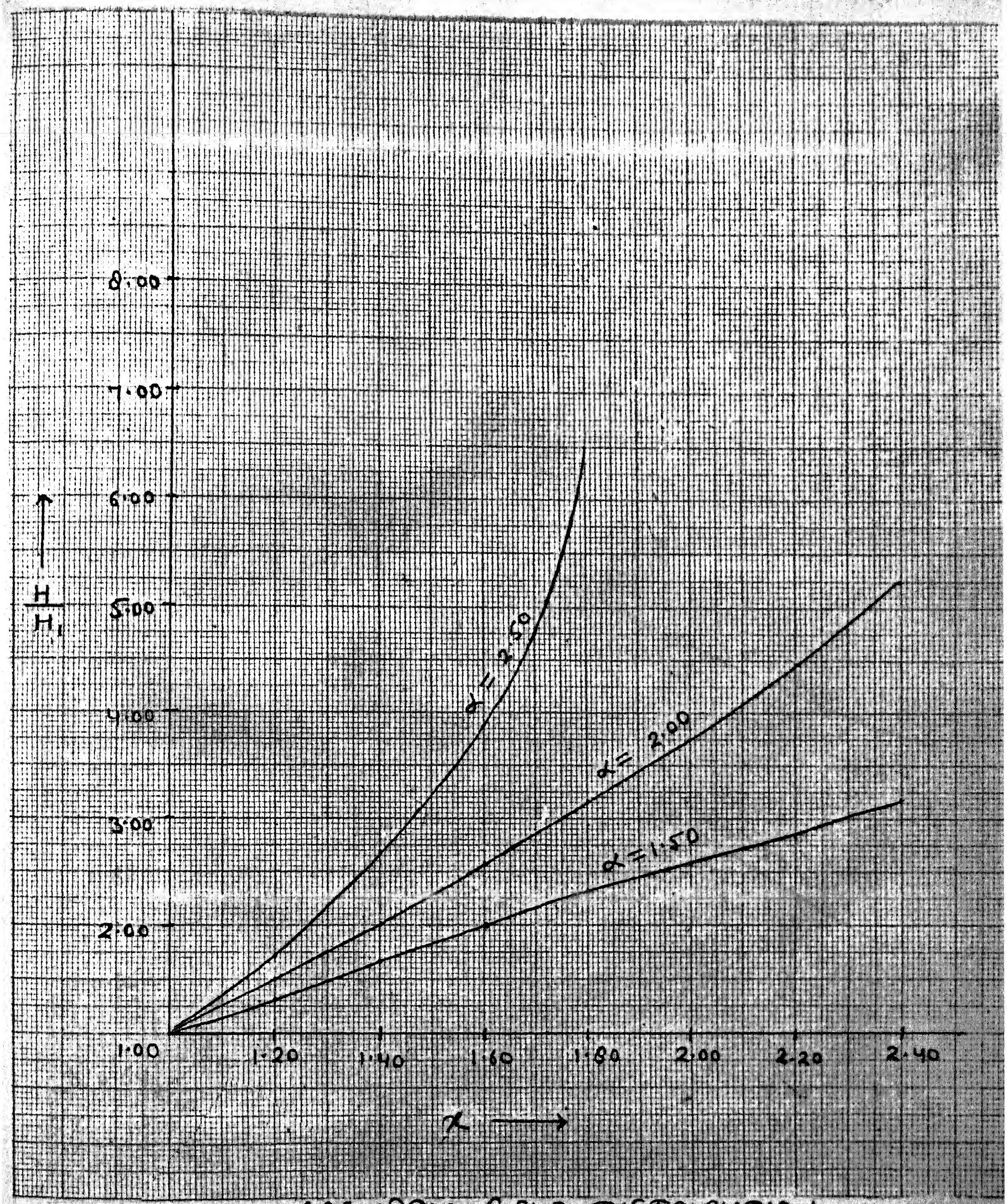
VELOCITY DISTRIBUTION



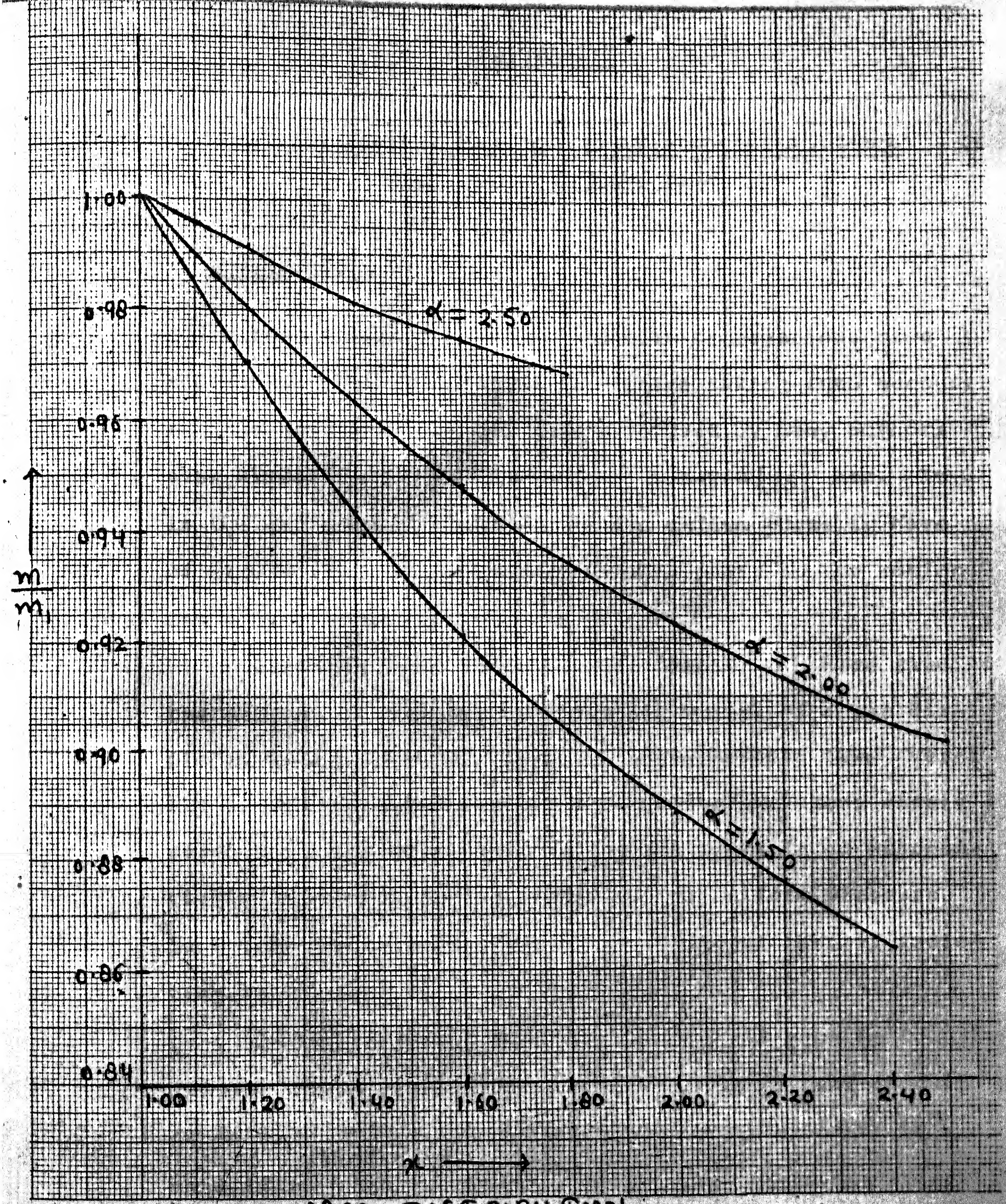
DENSITY DISTRIBUTION



PRESSURE DISTRIBUTION



MAGNETIC FIELD DISTRIBUTION



MASS DISTRIBUTION

CHAPTER-III

A SELF-SIMILAR FLOW BEHIND AN EXPONENTIAL SHOCK WITH RADIATIVE HEAT FLUX

1. INTRODUCTION

The problem of the self-similar propagations of a shock wave in a exponential atmosphere taking into account radiant heat exchange has been discussed by many authors viz. Raizer [23], Zel'dovich [24], Hayes [25] etc. It is the exponential character of the medium (density have he exponential law behaviour) which imparts to the motion its special features.

Gusev [26] and Ranga Rao and Ramana [27] have studied the problem of unsteady self-similar motion of a gas displaced by a piston according to an exponential law. Verma and Singh [28] and Singh and Shrivastava [29] have considered the problems of spherical shock waves in an exponentially increasing medium under the low uniform pressure including the effects of magnetic field and thermal radiation, respectively.

In this paper, a self-similar flow of a perfect inviscid gas behind a spherical shock propagating into an uniform exponential medium (which is assumed to be at rest) is considered. The surface of contact discontinuity moves with time according to an exponential law. The similarity solutions have been obtained taking radiative flux into consideration. Radiation pressure, radiant energy, solar gravity and conductivity have been neglected. It is assumed that

gas is opaque and shock is transparent.

The total energy of the wave varies as the cube of the shock radius.

2. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

For inviscid perfect gas, the differential equations governing the one-dimensional unsteady flow with radiation flux, behind a spherical shock are,

$$\frac{d\rho}{dt} + \frac{\rho}{r^2}\frac{\partial}{\partial r}(ur^2) = 0 , \tag{3.1}$$

$$\frac{du}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 , \tag{3.2}$$

$$\frac{dp}{dt} - \frac{\gamma p}{\rho}\frac{d\rho}{dt} + \frac{(\gamma-1)}{r^2}\frac{\partial}{\partial r}(r^2 F) = 0 , \tag{3.3}$$

and

$$p = \rho \Gamma T ; \tag{3.4}$$

where Γ being the gas constant and F denotes the heat flux.

Assuming local thermodynamic equilibrium, and taking Rosseland's diffusion approximation, which is,

$$F = -\frac{c\mu}{3}\frac{\partial}{\partial r}(\sigma T^4) \tag{3.5}$$

where $\frac{c\sigma}{4}$ is the Stefan-Boltzman constant, μ the mean-free path of radiation.

Following Wang [30] , μ which is a function of temperature and density, is

$$\mu = \mu_0 \rho^\alpha T^\beta . \tag{3.6}$$

μ_0 , α and β are constants.

The surface of contact discontinuity moves with time according to an exponential law

$$\bar{r} = A \exp(mt) \qquad (m > 0) \tag{3.7}$$

and since the flow is self-similar, the shock will also move with time according to an exponential law

$$R = B \exp(mt) \tag{3.8}$$

where $\bar{r}$ is the radius of inner expanding surface, R is the shock radius, A, B, and m are dimensional constants.

The shock boundary conditions are,

$$u_1 = \left[1 - \frac{1}{\gamma M^2}\right] v, \tag{3.9}$$

$$\rho_1 = \gamma M^2 \rho_0 \quad , \tag{3.10}$$

$$p_1 = \rho_0 v^2 \quad , \tag{3.11}$$

$$F_1 = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right] \rho_0 v^3 \; ; \tag{3.12}$$

Where subscripts 0 and 1 denote the region just ahead and behind the shock, respectively, and v is the fluid velocity.

3. SIMILARITY TRANSFORMATIONS

The similarity transformations for the problem under consideration are,

$$\eta = \frac{r}{B \exp(mt)} \; , \tag{3.13}$$

$$u = v\,V(\eta) \quad , \qquad (3.14)$$

$$\rho = \rho_0\,G(\eta) \quad , \qquad (3.15)$$

$$p = \rho_0 v^2 P(\eta) \quad , \qquad (3.16)$$

$$F = \rho_0 v^3 Q(\eta) \quad . \qquad (3.17)$$

The variable η assumed the values 1 and $\bar{\eta}$ at the shock and on the inner expanding surface, respectively.

So from (3.8) and (3.13) it follows that, the radius of the inner expanding surface

$$\bar{r} = \bar{\eta} R$$

also the variable

$$\eta = \frac{r}{R} \qquad (3.18)$$

as $\eta = 1$ at r = R from (3.13) we have

$$R = B \exp(mt)$$

$$\therefore \frac{dR}{dt} = mB \exp(mt)$$

or, the fluid velocity,

$$v = mR \,. \qquad (3.19)$$

By (3.5),

$$F = -\frac{c\mu}{3} \frac{\partial}{\partial r}(\sigma T^4)$$

using (3.4) and (3.6),

$$F = -\frac{\sigma c \mu_0}{3} \rho^{\alpha} \left(\frac{p}{T\rho}\right)^{\beta} \frac{\partial}{\partial r}\left(\frac{p^4}{T^4 \rho^4}\right)$$

$$F = -\frac{\sigma c \mu_0}{3\,T^{4+\beta}} \rho^{\alpha-\beta} p^{\beta} \frac{\partial}{\partial r}\left(\frac{p^4}{\rho^4}\right).$$

using (3.15), (3.16) and (3.17) we get

$$f_0 v^3 Q(\eta) = -\frac{\sigma c \mu_0}{3T^{4+\beta}} f_0^{\alpha} v^{2\beta+8} G(\eta)^{\alpha-\beta} P(\eta)^{\beta} \frac{\partial}{\partial r}\left[\frac{P^4(\eta)}{G^4(\eta)}\right]$$

or,

$$Q(\eta) = -\frac{\sigma c \mu_0}{3T^{4+\beta}} f_0^{\alpha-1} v^{2\beta+5} \frac{\partial \eta}{\partial r} G^{\alpha-\beta} P^{\beta}\left[\frac{4G^4P^3P' - 4P^4G^3G'}{G^8}\right]\frac{\partial \eta}{\partial r}$$

$$Q(\eta) = -\frac{4\sigma c \mu_0}{3T^{4+\beta}} f_0^{\alpha-1} v^{2\beta+5} \frac{\partial \eta}{\partial r} G^{\alpha-\beta-4} P^{\beta+4}\left[\frac{P'}{P} - \frac{G'}{G}\right]$$

using (3.18) and (3.19)

$$Q = -\frac{4\sigma c \mu_0 f_0^{\alpha-1} m}{3T^{4+\beta}} G^{(\alpha-\beta-4)} P^{(\beta+4)}\left[\frac{P'}{P} - \frac{G'}{G}\right]$$

$$Q = -N G^{(\alpha-\beta-4)} P^{(\beta+4)}\left[\frac{P'}{P} - \frac{G'}{G}\right] \tag{3.20}$$

with $\beta = -2$, α remaining arbitrary, ($0 \leq \alpha \leq 2$) and

$$N = \frac{4\sigma m c \mu_0 f_0^{\alpha-1}}{3T^{4+\beta}}$$

= a dimensionless radiation parameter.

4. SOLUTIONS OF EQUATIONS OF MOTION

The equations (3.13) to (3.19) are used to reduce the equations (3.1) to (3.3) and (3.20) in a new form.

$$f = f_0 G(\eta)$$

$$\frac{\partial f}{\partial t} = f_0 G'(\eta) \cdot \frac{\partial \eta}{\partial t}$$

$$= f_0 G'(\eta) \cdot -\frac{m r}{R} \qquad \text{as } \frac{\partial \eta}{\partial t} = -\frac{m r}{R}$$

$$\frac{\partial \rho}{\partial t} = -m\rho_0 \eta G'(\eta)$$

also

$$\frac{\partial \rho}{\partial r} = \rho_0 G'(\eta) \cdot \frac{\partial \eta}{\partial r}$$

$$= \frac{\rho_0 G'(\eta)}{R}$$

$$\therefore u\frac{\partial \rho}{\partial r} = \frac{u\rho_0}{R} G'(\eta) V(\eta)$$

$$u\frac{\partial \rho}{\partial r} = m\rho_0 G'(\eta) V(\eta)$$

$$u = \upsilon V(\eta)$$

so,

$$\frac{\partial u}{\partial r} = \frac{\upsilon V'(\eta)}{R} \qquad \text{as} \quad \frac{\partial \eta}{\partial r} = \frac{1}{R}$$

$$= m V'(\eta)$$

$$\therefore \rho\frac{\partial u}{\partial r} = m\rho_0 V'(\eta) G(\eta)$$

and,

$$\frac{2\rho u}{r} = \frac{2\rho_0 \upsilon G(\eta) V(\eta)}{\eta R}$$

$$= \frac{2m\rho_0}{\eta} G(\eta) V(\eta)$$

Substituting these values in (3.1)

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{2\rho u}{r} + \rho\frac{\partial u}{\partial r} = 0$$

$$-m\rho_0\eta G'(\eta) + m\rho_0 G'(\eta)V(\eta) + \frac{2m\rho_0}{\eta} G(\eta)V(\eta) + m\rho_0 V'(\eta)G(\eta) = 0$$

or,

$$-\eta G' + G'V + \frac{2}{\eta} GV + V'G = 0$$

or

$$G'(\eta - V) = \frac{G}{\eta}(2V + \eta V')$$

$$G' = \frac{G(2V + \eta V')}{\eta(\eta - V)} \tag{3.21}$$

Equation (3.2) is,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0$$

$$u = v\,V(\eta).$$

$$\therefore \frac{\partial u}{\partial t} = v\,V'(\eta)\frac{\partial \eta}{\partial t} + V(\eta)\frac{\partial v}{\partial t}$$

since, $\quad v = mR = mBe^{mt}$

$$\frac{\partial v}{\partial t} = m^2 R$$

and

$$\frac{\partial \eta}{\partial t} = -\frac{mr}{R} = -m\eta$$

$$\frac{\partial u}{\partial t} = -mv\eta V'(\eta) + mv\,V(\eta)$$

and,

$$\frac{\partial u}{\partial r} = \frac{v\,V'(\eta)}{R}$$

$$\therefore u\frac{\partial u}{\partial r} = \frac{v^2}{R}V'(\eta)\,V(\eta)$$

$$u\frac{\partial u}{\partial r} = mv\,V'(\eta)\,V(\eta)$$

$$p = \rho_0 v^2 P(\eta) \qquad \text{(By 3.16)}$$

$$\frac{\partial p}{\partial r} = \rho_0 v^2 P'(\eta)\frac{\partial \eta}{\partial r}$$

$$= \frac{\rho_0 v^2 P'(\eta)}{R}$$

$$\therefore \frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{\rho_0 v^2 P'(\eta)}{\rho_0 G(\eta).R}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = mv\frac{P'(\eta)}{G(\eta)}$$

Substituting these values in (3.2)

$$-m\upsilon\eta V'(\eta) + m\upsilon V(\eta) + m\upsilon V'(\eta)V(\eta) + m\upsilon \frac{P'(\eta)}{G(\eta)} = 0$$

or

$$-\eta V' + V + V'V + \frac{P'}{G} = 0$$

or

$$\frac{P'}{G} = \eta V' - V - V'V$$

$$P' = G\left[V'(\eta - V) - V\right] \tag{3.22}$$

Equation (3.3) is,

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - \frac{\gamma p}{f}\frac{\partial f}{\partial t} - \frac{\gamma p u}{f}\frac{\partial f}{\partial r} + \frac{\gamma - 1}{r^2}\frac{\partial}{\partial r}(r^2 F) = 0$$

$$p = f_0 \upsilon^3 P(\eta)$$

$$\frac{\partial p}{\partial t} = f_0\left[2\upsilon\frac{\partial \upsilon}{\partial t}P(\eta) - \upsilon^2 P'(\eta)\frac{\partial \eta}{\partial t}\right]$$

$$= f_0\left[2\upsilon m^2 R\, P(\eta) - m\upsilon^2\eta\, P'(\eta)\right]$$

$$\frac{\partial p}{\partial t} = m\upsilon^2 f_0\left[2P(\eta) - \eta P'(\eta)\right]$$

substituting the value of P' from (3.22),

$$\frac{\partial p}{\partial t} = m\upsilon^2 f_0\left[2P - \eta G(\eta - V)V' - \eta G V\right]$$

$$\frac{\partial p}{\partial r} = \frac{f_0 \upsilon^2 P'(\eta)}{R}$$

$$\therefore\ u\frac{\partial p}{\partial r} = \frac{f_0 \upsilon^3 P'(\eta) V(\eta)}{R}$$

$$u\frac{\partial p}{\partial r} = m f_0 \upsilon^2 V\left[G(\eta - V)V' - GV\right]$$

and,

$$\frac{\gamma p}{f}\frac{\partial f}{\partial t} = -\frac{\gamma v^2 P(\eta)}{G(\eta)} \cdot m f_0 \eta G'(\eta)$$

substituting the value of $\frac{G'(\eta)}{G(\eta)}$ from (3.21)

$$\frac{\gamma p}{f}\frac{\partial f}{\partial t} = -m f_0 v^2 \eta \frac{P(2V + \eta V')}{(\eta - V)}$$

$$\frac{\partial f}{\partial r} = \frac{f_0 G'(\eta)}{R}$$

so,

$$\frac{\gamma p u}{f}\frac{\partial f}{\partial r} = \frac{f_0 \gamma v^2 P(\eta) \cdot v V(\eta)}{R G(\eta)} \cdot G'(\eta)$$

$$= m f_0 v^2 \eta \frac{PV(\eta V' + 2V)}{\eta(\eta - V)}$$

$$\frac{\partial}{\partial r}(r^2 F) = \frac{\partial}{\partial r}\left[r^2 f_0 v^3 Q(\eta)\right]$$

$$= f_0 v^3 \left[2r Q(\eta) + r^2 Q'(\eta)\frac{\partial \eta}{\partial r}\right]$$

$$= f_0 v^3 r \,(2Q + \eta Q')$$

$$\therefore \frac{(\gamma-1)}{r^2}\frac{\partial}{\partial r}(r^2 F) = \frac{f_0 r v^3 (\gamma - 1)}{r^2}\left[2Q(\eta) + \eta Q'(\eta)\right]$$

$$= \frac{m f_0 R v^2 (\gamma-1)}{\eta R}\left[2Q(\eta) + \eta Q'(\eta)\right]$$

$$\frac{(\gamma-1)}{r^2}\frac{\partial}{\partial r}(r^2 F) = m f_0 v^2 \frac{(\gamma-1)}{\eta}(2Q + \eta Q')$$

Substituting these values in (3.3),

$$m U^2 \rho_0 \left[2P(\eta) - \eta G(\eta)\left(\eta - V(\eta)\right) V'(\eta) + \eta G(\eta) V(\eta) \right] +$$

$$+ \; m \rho_0 U^2 V(\eta) \left[G(\eta)\left(\eta - V(\eta)\right) V'(\eta) - G(\eta) V(\eta) \right] +$$

$$+ \; m \rho_0 U^2 \gamma \frac{P(\eta)\left[\eta V'(\eta) + 2V(\eta)\right]}{\left[\eta - V(\eta)\right]} - m \rho_0 U^2 \gamma \frac{P(\eta) V(\eta)\left[\eta V'(\eta) + 2V(\eta)\right]}{\eta\left[\eta - V(\eta)\right]} +$$

$$+ \; m \rho_0 U^2 \frac{(\gamma - 1)}{\eta}\left[2Q(\eta) + \eta Q'(\eta)\right] = 0 .$$

Or,

$$2P - \eta G(\eta - V) V' + \eta G V + V G(\eta - V) V' - V^2 G +$$

$$+ \frac{\gamma P(\eta V' + 2V)}{(\eta - V)} - \frac{P V(\eta V' + 2V)}{\eta(\eta - V)} +$$

$$+ \frac{(\gamma - 1)}{\eta}(2Q + \eta Q') = 0$$

Or,

$$2P\eta(\eta - V) - \eta^2 G(\eta - V)^2 + \eta^2(\eta - V) G V + \eta V G(\eta - V)^2 V' -$$

$$- \eta(\eta - V) V^2 G + \gamma \eta^2 P V' + 2\gamma \eta P V - \gamma \eta P V V' - 2\gamma P V^2 +$$

$$+ 2(\gamma - 1) Q(\eta - V) + \eta(\gamma - 1) Q'(\eta - V) = 0$$

Or,

$$2P\eta(\eta - V) - G V' \eta (\eta - V)^2 (\eta - V) + \eta(\eta - V) G V(\eta - V) +$$

$$+ \eta \gamma P V'(\eta - V) + 2\gamma P V(\eta - V) + 2(\gamma - 1)(\eta - V) Q +$$

$$+ \eta(\gamma - 1) Q'(\eta - V) = 0$$

or,

$$2P\eta - Gv'\eta(\eta-v)^2 + Gv\eta(\eta-v) + \gamma\eta Pv' + 2\gamma Pv +$$
$$+ 2(\gamma-1)Q + \eta(\gamma-1)Q' = 0$$

or,

$$-G\eta\left[(\eta-v)^2 v' - v(\eta-v) - \frac{\gamma P}{G}v'\right] + 2P(\eta+\gamma v) +$$
$$+ 2(\gamma-1)Q + \eta(\gamma-1)Q' = 0$$

or,

$$\eta(\gamma-1)Q' = G\eta\left[v'\left\{(\eta-v)^2 - \frac{\gamma P}{G}\right\} - v(\eta-v)\right] - 2P(\eta+\gamma v) -$$
$$- 2(\gamma-1)Q$$

$$Q' = \frac{\left\langle G\eta\left[v'\left\{(\eta-v)^2 - \frac{\gamma P}{G}\right\} - v(\eta-v)\right] - 2P(\eta+\gamma v) - 2(\gamma-1)Q\right\rangle}{\eta(\gamma-1)} \qquad (3.23)$$

Equation (3.20) is,

$$Q = -NG^{(\alpha-\beta-4)}\,P^{(\beta+4)}\left(\frac{P'}{P} - \frac{G'}{G}\right)$$

since, $\beta = -2$

$$Q = -NG^{(\alpha-1)-1}\,P^2\left(\frac{P'}{P} - \frac{G'}{G}\right)$$

$$Q = -NG^{(\alpha-1)}\,\frac{P^2}{GP}\left(P' - \frac{PG'}{G}\right)$$

Substituting the value of P' and $\frac{G'}{G}$ from (3.21) and (3.22),

$$Q = -NG^{(\alpha-1)}\,\frac{P}{G}\left[G\left\{v'(\eta-v) - v\right\} - \frac{P\eta v' + 2Pv}{\eta(\eta-v)}\right]$$

$$Q = -NG^{(\alpha-1)}P\left[V'(\eta-V) - V - \frac{(P/G)\eta V' + 2(P/G)V}{\eta(\eta-V)}\right]$$

or

$$-\frac{QG^{(1-\alpha)}}{NP} = (\eta-V)V' - V - \frac{(P/G)\eta V'}{\eta(\eta-V)} - \frac{2(P/G)V}{\eta(\eta-V)}$$

or

$$-\frac{QG^{(1-\alpha)}}{NP}\eta(\eta-V) = \eta(\eta-V)^2V' - \eta(\eta-V)V - (P/G)\eta V' - 2(P/G)V$$

or

$$\eta V'\left[(\eta-V)^2 - (P/G)\right] = 2(P/G)V + \eta(\eta-V) - \frac{QG^{(1-\alpha)}}{NP}\eta(\eta-V)$$

$$= 2(P/G)V - \eta(\eta-V)\left[\frac{QG^{(1-\alpha)}}{NP} - V\right]$$

or

$$V' = \frac{2V(P/G) - \eta(\eta-V)\left[\frac{QG^{(1-\alpha)}}{NP} - V\right]}{\eta\left[(\eta-V)^2 - P/G\right]} \tag{3.24}$$

The shock conditions (3.9) to (3.12) are transformed into the following forms, (at $\eta = \eta_0$)

Boundary condition (3.9) is,

$$u_1 = \left[1 - \frac{1}{\gamma M^2}\right]\dot{U}$$

$$\dot{U}\,V(\eta) = \left[1 - \frac{1}{\gamma M^2}\right]\dot{U}$$

$\eta = 1$ at r = R so,

$$V(1) = \left[1 - \frac{1}{\gamma M^2}\right] \tag{3.25}$$

Condition (3.10) is,

$$\rho_1 = \gamma M^2 \rho_0$$

$$\rho_0 G(\eta) = \gamma M^2 \rho_0 \qquad \text{using (3.15)}$$

at $\eta = 1$,

$$G(1) = \gamma M^2 \tag{3.26}$$

Condition (3.11) is,

$$p_1 = \rho_0 v^2$$

$$\rho_0 v^2 P(\eta) = \rho_0 v^2$$

at $\eta = 1$,

$$P(1) = 1 \tag{3.27}$$

and condition (3.12) is,

$$F_1 = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right] \rho_0 v^3$$

$$\rho_0 v^3 Q(\eta) = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right] \rho_0 v^3$$

at $\eta = 1$

$$Q(1) = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right] \tag{3.28}$$

Thus the transformed equations of motion are,

$$G' = \frac{G(\eta V' + 2V)}{\eta(\eta - V)},$$

$$P' = G\left[(\eta - V)V' - V\right],$$

$$Q' = \frac{\left\langle G\eta\left[V'\left\{(\eta-V)^2 - \gamma P/G\right\} - V(\eta-V)\right] - 2P(\eta+\gamma V) - 2(\gamma-1)Q\right\rangle}{\eta(\gamma-1)},$$

and

$$V' = \frac{2VP/G - \eta(\eta-V)\left[Q G^{(1-\alpha)}/NP - V\right]}{\eta\left[(\eta-V)^2 - P/G\right]}.$$

And appropriate transformed shock conditions are,

$$V(1) = \left[1 - \frac{1}{\gamma M^2}\right],$$

$$G(1) = \gamma M^2,$$

$$P(1) = 1,$$

and

$$Q(1) = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right];$$

where prime denotes differentiation with respect to η

5. NUMERICAL RESULTS FOR THE SOLUTIONS

For numerical calculation, it is convenient to write the flow variables in the following non-dimensional forms, (using (3.14) to (3.17) and (3.25) to (3.28))

$$\frac{u}{u_1} = \frac{v V}{\left[1 - \frac{1}{\gamma M^2}\right] v}$$

$$= \frac{V}{\left[1 - \frac{1}{\gamma M^2}\right]}.$$

$$\frac{u}{u_1} = \frac{V}{V(1)}, \tag{3.29}$$

$$\frac{\rho}{\rho_1} = \frac{\rho_0 G}{\gamma M^2 \rho_0}$$

$$\frac{\rho}{\rho_1} = \frac{G}{G(1)} , \tag{3.30}$$

$$\frac{p}{p_1} = \frac{\rho_0 U^2 P}{\rho_0 U^2}$$

$$\frac{p}{p_1} = \frac{P}{P(1)} , \tag{3.31}$$

and
$$\frac{F}{F_1} = \frac{\rho_0 U^3 Q}{\frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right] \rho_0 U^3}$$

$$\frac{F}{F_1} = \frac{Q}{Q(1)} ; \tag{3.32}$$

The numerical integration was carried out until the kinematic condition $V(\eta) = \eta$ is satisfied and was performed on a computer, using the Runge-Kutta programme for N = 10 and N = 100. The other constants are M = 20, $\gamma = 1.4$ and $\alpha = 1$. The nature of the flow variables is given in tables I and II for N = 10 and 100, respectively.

6. <u>CONCLUSION</u>

In the problem of the propagation of an exponential shock wave with radiative heat flux, there is very minute variation in velocity throughout the medium, while the variations in pressure, density and heat flux are very minute in the beginning (near the shock surface) and then rapid in the last (apart from the shock surface). For both the cases N = 10 and 100 the velocity, density and pressure

are minimum at the shock surface (i.e. at $\eta = 1$) and increase on moving toward the point of explosion from the shock surface whereas the value of heat flux is maximum at shock surface and decreases on moving toward the point of explosion from the shock surface.

TABLE 1
$N = 10$

η	u/u_1	ρ/ρ_1	p/p_1	F/F_1
1.00	1.0000	1.00000	1.00000	1.00000
0.98	1.02160	1.01921	0.97718	0.578678
0.96	1.04273	1.04060	0.96639	0.19186
0.94	1.06344	1.06421	0.96739	−0.16243
0.92	1.08377	1.09014	0.97969	−0.48817
0.90	1.10378	1.11848	1.00264	−0.79079
0.88	1.12352	1.14934	1.03558	−1.07671
0.86	1.14103	1.18288	1.07790	−1.35277
0.84	1.16235	1.12927	1.12914	−1.62588
0.82	1.18151	1.25873	1.18904	−1.90287
0.80	1.20052	1.30150	1.25752	−2.19037
0.78	1.21942	1.34789	1.33471	−2.49495
0.76	1.23819	1.39824	1.42095	−2.82325
0.74	1.25687	1.45293	1.51676	−3.18215
0.72	1.27543	1.51243	1.62288	−3.579023
0.70	1.29390	1.57725	1.74025	−4.021976
0.68	1.31227	1.64799	1.87005	−4.52014
0.66	1.33054	1.72537	2.01267	−5.08403
0.64	1.34870	1.81019	2.17280	−5.72591
0.62	1.36675	1.90339	2.34946	−6.46028
0.60	1.38468	2.00607	2.54601	−7.30450
0.58	1.40249	2.11955	2.76528	−8.27950
0.56	1.42017	2.24534	3.01065	−9.41077
0.54	1.43771	2.38528	3.28612	−10.72959
0.52	1.45511	2.54155	3.59653	−12.27459
0.50	1.47234	2.71676	3.94771	−14.09393
0.48	1.48940	2.914105	4.34674	−16.24810
0.46	1.506288	3.13746	4.80228	−18.81374
0.44	1.52296	3.39161	5.32502	−21.88883
0.42	1.53942	3.62851	5.92825	−25.59990
0.40	1.55565	4.01763	6.62867	−30.11209
0.38	1.57161	4.40646	7.44743	−35.64345
0.36	1.58729	4.86118	8.41170	−42.48563
0.34	1.60265	5.39767	9.55677	−51.034227
0.32	1.61766	6.036942	10.92913	−61.83454
0.30	1.63227	6.80719	12.59102	−75.65102
0.28	1.64645	7.747108	14.62720	−93.57737
0.26	1.66013	8.21080	17.15552	−117.21341
0.24	1.673241	10.37590	20.34351	−148.96023
0.22	1.68569	12.25699	24.43582	−192.52994
0.20	1.697380	14.72889	29.80097	−253.86424
0.18	1.70816	18.06907	37.01473	−342.87355
0.16	1.71786	22.73964	47.01767	−476.93058
0.14	1.72623	29.55685	61.43244	−668.4145
0.12	1.73294	40.07475	83.25880	−1042.5423
0.10	1.737484	57.55726	118.5668	−1684.7271
0.08	1.739091	89.85132	181.2596	−2987.7993
0.06	1.73664	159.9835	309.5294	−6126.2032
0.04	1.72688	361.9695	644.8004	−16303.104
0.02	1.70351	1467.9465	2160.4617	−80996.104
0.007	1.67116	9727.8308	10319.152	−639120.97

TABLE II
$N = 100$

η	u/u_1	ρ/ρ_1	p/p_1	F/F_1
1.00	1.00000	1.00000	1.00000	1.00000
0.98	1.02084	1.01997	0.99728	0.67336
0.96	1.04161	1.04171	0.99635	0.33083
0.94	1.06232	1.06533	0.99728	−0.02956
0.92	1.08297	1.09095	1.00017	−0.40831
0.90	1.10355	1.11871	1.00514	−0.80858
0.88	1.12407	1.14878	1.01231	−1.12321
0.86	1.14454	1.18133	1.02184	−1.68149
0.84	1.16494	1.21656	1.03391	−2.18413
0.82	1.18529	1.24571	1.04870	−2.66918
0.80	1.20559	1.29604	1.06643	−3.31444
0.78	1.22583	1.34085	1.08735	−3.79946
0.76	1.24601	1.38946	1.11173	−4.42914
0.74	1.26615	1.44228	1.13989	−5.10918
0.72	1.28623	1.49974	1.17217	−5.84617
0.70	1.30625	1.56233	1.20899	−6.64782
0.68	1.32623	1.63065	1.25079	−7.52315
0.66	1.34616	1.70535	1.29808	−8.48276
0.64	1.36603	1.78722	1.35147	−9.53912
0.62	1.38586	1.87714	1.41164	−10.7070
0.60	1.40563	1.97618	1.47938	−12.0040
0.58	1.42535	2.08555	1.55562	−13.4511
0.56	1.44502	2.20673	1.64144	−15.0735
0.54	1.46464	2.34143	1.73813	−16.9017
0.52	1.48420	2.49172	1.84719	−18.9727
0.50	1.50371	2.66008	1.97045	−21.3316
0.48	1.52317	2.84961	2.11007	−24.0344
0.46	1.54256	3.06367	2.26870	−27.1501
0.44	1.56190	3.30706	2.44956	−30.7657
0.42	1.58117	3.58529	2.65660	−34.9907
0.40	1.60038	3.90535	2.89475	−39.9654
0.38	1.61951	4.27613	3.17019	−45.8708
0.36	1.63857	4.70905	3.49075	−52.9440
0.34	1.65754	5.21892	3.86650	−61.4993
0.32	1.6743	5.82530	4.31055	−71.9600
0.30	1.69522	6.55445	4.84021	−84.9064
0.28	1.71390	7.44224	5.47879	−101.1491
0.26	1.73246	8.53877	6.25823	−121.845
0.24	1.75089	9.91573	7.22333	−148.689
0.22	1.76917	11.7786	8.43858	−184.237
0.20	1.78728	13.9880	9.99963	−232.471
0.18	1.80518	17.0799	12.0536	−299.860
0.16	1.82285	21.4299	14.83682	−397.429
0.14	1.84023	27.7259	18.7498	−545.143
0.12	1.85726	37.3923	24.5193	−781.998
0.10	1.87383	53.3690	33.5857	−1192.179
0.08	1.88981	82.6822	49.1757	−1984.135
0.06	1.90493	145.832	79.9317	−3789.765
0.04	1.91870	325.784	156.921	−9287.3470
0.02	1.92980	1295.814	485.751	−41556.363
0.007	1.93312	10565.264	2577.718	−379451.04

REFERENCES

[1] P.D. McCormack & Lawrence Crane (1973) Physical Fluid Dynamics. Academic Press INC., New York.

[2] J.K. Goyal (1986) Fluid Dynamics. Pragati Prakashan, Meerut.

[3] R. Von Mises (1958) Mathematical Theory of Compressible Fluid flow. Academic Press INC., New York.

[4] Frank Chorlton (1985) Text-book of Fluid Dynamics. CBS Publishers, New Delhi.

[5] S.I. Pai (1959) Introduction to the Theory of Compressible Flow. D.Von Nostrand Coy., INC, New York.

[6] A.K. Mohanty (1986) Fluid Mechanics. Prentice-Hall of India Pvt. Ltd., New Delhi.

[7] G.B. Whitham (1974) Linear and Non-linear waves. A Wiley-Interscience Pub.

[8] R. Von Mises & K.O. Friedrichs (1971) Fluid Dynamics. Springer-Varlag, INC., New York.

[9] F. de Hoffmann & E. Teller (1950) Magnetohydrodynamic Shocks. Phy. Rev., 80, 692.

[10] B. Chakraborty (1978) Principle of Plasma Mechanics, Chapter four. National Book Trust, India.

[11] J.D. Jackson (1975) Classical Electrodynamics. John Wiley and Sons, INC., New York.

[12] S. Chandrashekhar (1957) An Introduction to the Study of Stellar Structure. Dover Publication INC., New York.

[13] Ya. B. Zel'dovich & Yu. P. Reizer (1966) Physics of Shock Waves and High Temperature Hydrody. Phenomena. Vol. I, Acadamic Press, New York.

[14] Landau & Lifshitz (1959) Fluid Mechanics. Pergamon Press, New York.

[15] Ya. B. Zel'dovich & Yu. P. Reizer (1967) Physics of Shock Waves and High Temperature Hydrody. Phenomena. Vol. II, Acadamic Press, New York.

[16] L.I. Sedov (1959) Similarity and Dimensional Methods in Mechanics. Acadamic Press, New York.

[17] K.P. Stankukovich (1960) Unsteady motion of continuous Media. Academic Press, New York.

[18] G.I. Taylor (1950) The Formation of a Blast Wave by a very Intense Explosion. II. The Atomic Explosion of 1945, Proc. Roy. Soc.(London), Ser. A 201, 175-186.

[19] P. Carrus, P. Fox, F. Haas & Z.Kopal. (1951) Astrophys. J. 125, 495.

[20] E.V.Ryazanov (1959) Soviet Phys. Dokl. SSSR 4, 537.

[21] J.B.Singh (1982) Astrophys. Space Sci. 87, 347.

[22] G.B. Whitham (1958) J. Fluid Mech. 4, 337.

[23] Yu. P. Raizer (1957) On the Structure of the Front of Strong Shock Waves in gases, Soviet Phys. JETP (English Translation) 5, 1242-1248.

[24]	Ya.B.Zel'dovich & Yu. P. Raizer.	(1957)	Strong Shock Waves in gases, VST. Fiz. Nauk 63,613-641.
[25]	W.D. Hayes		Self-Similar Strong Shocks in an Exponential Medium. J. Fluid Mech., (to appear)
[26]	V.N. Gusev	(1957)	TSAGI Rep. BNI.
[27]	M.P. Ranga Rao & B.V. Ramana	(1976)	J. Math. Phys. Sct.10, 465.
[28]	B.G. Verma & J.B. Singh	(1979)	Astrophys. Space Sci.63,253.
[29]	J.B. Singh & S.K. Srivastava	(1982)	Astrophys. Space Sci.88,277.
[30]	K.C. Wang	(1966)	Phys. Fluids 9, 1922.

A SELF-SIMILAR FLOW OF SELF-GRAVITATING GAS BEHIND A SPHERICAL SHOCK WAVE IN MAGNETOGASDYNAMICS

†B. G. VERMA* and V. K. SINGH**
Department of Mathematics, University of Gorakhpur, India

(Received 7 October, 1985)

Abstract. This paper considers a spherical shock, in a conducting gas, of self-gravitating gas propagating in a non-uniform atmosphere at rest. Similarity principle has been used to reduce the equations governing the flow to ordinary differential equations under the assumption that the density varies as an inverse-power of distance from the point or explosion. The total energy of the wave is variable.

1. Introduction

The unsteady motion of a large mass of gas followed by sudden release of energy results in novae and supernovae. The explanation and analysis of the internal motion in stars is one of the basic problems in astrophysics. Numerical solutions for self-similar adiabatic flow in self-gravitating gas were obtained by Carrus *et al.* (1951) and Sedov (1959). Starting with these results, Ryazanov (1959) obtained a particular analytic solution. But this solution does not describe the flow behaviour in general. Singh (1982) has discussed the self-similar adiabatic flow of self-gravitating gas in ordinary gas dynamics and has obtained numerical solutions.

In this paper we show that the magnetic field has a significant effect on the physical parameters when the gas is self-gravitating. The motion of the shock wave is assumed to satisfy the power law

$$\rho_0 = AR^{-\alpha}, \tag{1.1}$$

where A and α are constants and R is the shock radius. The total energy of the flow increases with time because of the pressure exerted on the gas by an expanding surface. Therefore,

$$E = Bt^q \quad (q \geq 0), \tag{1.2}$$

where B and q are constants and E is the total energy. The magnetic field distribution law is

$$h_0 = CR^{-\beta} \quad (\beta \geq 0), \tag{1.3}$$

where C and β are constants, and the values of q and β are to be determined later. The magnetic field is directed tangential to the advancing shock front.

* Professor Verma passed away on 4 April, 1985.
** Supported by CSIR, New Delhi under grant No. 7/57/287/81/EMR-I.

Astrophysics and Space Science **124** (1986) 53–62.

The flow variable just ahead of the shock are

$$u_0 = 0\,, \qquad m_0 = \frac{4\pi A}{3-\alpha}\, R^{3-\alpha}$$

and

$$p_0 = \frac{2\pi A^2 G}{(\alpha-1)(3-\alpha)}\, R^{2-2\alpha} + \frac{C^2}{2\beta}\,(1-\beta)R^{-2\beta}\,, \tag{1.4}$$

where

$$1 + \beta = \alpha \tag{1.5}$$

and viscosity is neglected.

We investigate three types of models, the first having total energy of the explosion to be constant ($\alpha = 2.5$), the second having constant velocity of propagation of shock waves ($\alpha = 2$) and the third having neither constant total energy nor constant velocity propagation of shock waves ($\alpha = 1.5$). An idealized magnetic field is considered for only a portion of sphere enclosing the origin – i.e., the point of explosion.

2. Equations of the Problem

The basic differential equations governing the adiabatic flow in a self-gravitating gas are given by

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\,\frac{\partial}{\partial r}\,(r^2 \rho u) = 0\,, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial r} + \frac{1}{\rho}\,\frac{\partial p}{\partial r} + \frac{H}{\rho}\,\frac{\partial H}{\partial r} + \frac{H^2}{\rho r} + \frac{Gm}{r^2} = 0\,, \tag{2.2}$$

$$\frac{\partial H}{\partial t} + u\,\frac{\partial H}{\partial r} + H\,\frac{\partial u}{\partial r} + \frac{HU}{r} = 0\,, \tag{2.3}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^{\gamma}}\right) + u\,\frac{\partial}{\partial r}\left(\frac{p}{\rho^{\gamma}}\right) = 0\,, \tag{2.4}$$

$$\frac{\partial m}{\partial r} = 4\pi \rho r^2\,, \tag{2.5}$$

where r, t, m, u, ρ, p, and γ are radial distance from the centre, time, mass contained in a sphere of radius γ, velocity, density, pressure, and ratio of two specific heats, respectively, and G represents the gravitational constant.

The Rankine–Hugoniot boundary conditions at the shock (cf. (2.5)) are

$$u_1 = \frac{\xi - 1}{\xi} V, \tag{2.6}$$

$$\rho_1 = \rho_0 \xi, \tag{2.7}$$

$$p_1 = \psi \rho_0 V^2, \tag{2.8}$$

$$H_1 = H_0 \xi, \tag{2.9}$$

$$m_1 = m_0; \tag{2.10}$$

where

$$\psi = \frac{1}{\gamma M^2} + \frac{2(\xi - 1)}{[(\gamma + 1) - (\gamma - 1)\xi]}\left[\frac{1}{M^2} + \frac{\gamma - 1}{4M_A^2}(\xi - 1)^2\right], \tag{2.11}$$

$V = \mathrm{d}R/\mathrm{d}t$ being the shock velocity and ξ is given by the quadratic equation

$$M_A^{-2}(2 - \gamma)\xi^2 + \left[(2\gamma - 1) + \frac{2}{M^2}\right]\xi - (\gamma + 1) = 0. \tag{2.12}$$

The Mach number M and Alfvén's Mach number M_A are given by

$$M^2 = \frac{V^2 \rho_0}{\gamma p_0} \quad \text{and} \quad M_A^2 = \frac{V^2 \rho_0}{H_0^2}. \tag{2.13}$$

Let us seek the solution to the equation in the following form

$$u = \frac{r}{t} U(\eta), \qquad \rho = r^K t^{\lambda} \overset{\Omega(\eta)}{(\cancel{\eta})}, \qquad m = r^{K+3} t^{\lambda} W(\eta), \tag{2.14}$$

$$p = r^{K+2} t^{\lambda - 2} P(\eta), \qquad H = r^{(K+2)/2}\, t^{(\lambda - 2)/2} N(\eta), \tag{2.15}$$

where $\eta = r^a t^b$; while K, λ, a, and b are constantse and are to be determined from the conditions of the problem. The total energy E inside the shock wave of radius R is given by

$$E = 4\pi \int_0^R \left(\frac{1}{2}\rho u^2 + \frac{p}{\gamma - 1} + \frac{h^2}{2} - \frac{Gm\rho}{r}\right) r^2\, \mathrm{d}r = Bt^q \quad (q \geq 0). \tag{2.16}$$

In terms of the variable η, we can express the total energy as

$$E = \frac{4\pi}{a} \int_{\eta^x}^{\eta_0} \left[\eta^{[(K+5)/a] - 1} \cancel{t}^{\lambda - 2 - (b/a)(K+5)} \left(\frac{1}{2} U^2 \Omega + \frac{P}{\gamma - 1} + N^2\right) - \right.$$

$$\left. - GW\Omega \eta^{[(2K+5)/a] - 1}\, t^{2\lambda - (b/a)(2K+5)}\right] \mathrm{d}\eta = Bt^q; \tag{2.17}$$

η_0 and η^x being the values of η at the shock front and expanding surface, respectively.

Equation (2.17) yields

$$\frac{a}{b} = \frac{-5}{4+q}. \tag{2.18}$$

We choose η_0 to be constant at the shock surface. This choice fixes the velocity of the shock as

$$V = -\frac{b}{a}\frac{R}{t}. \tag{2.19}$$

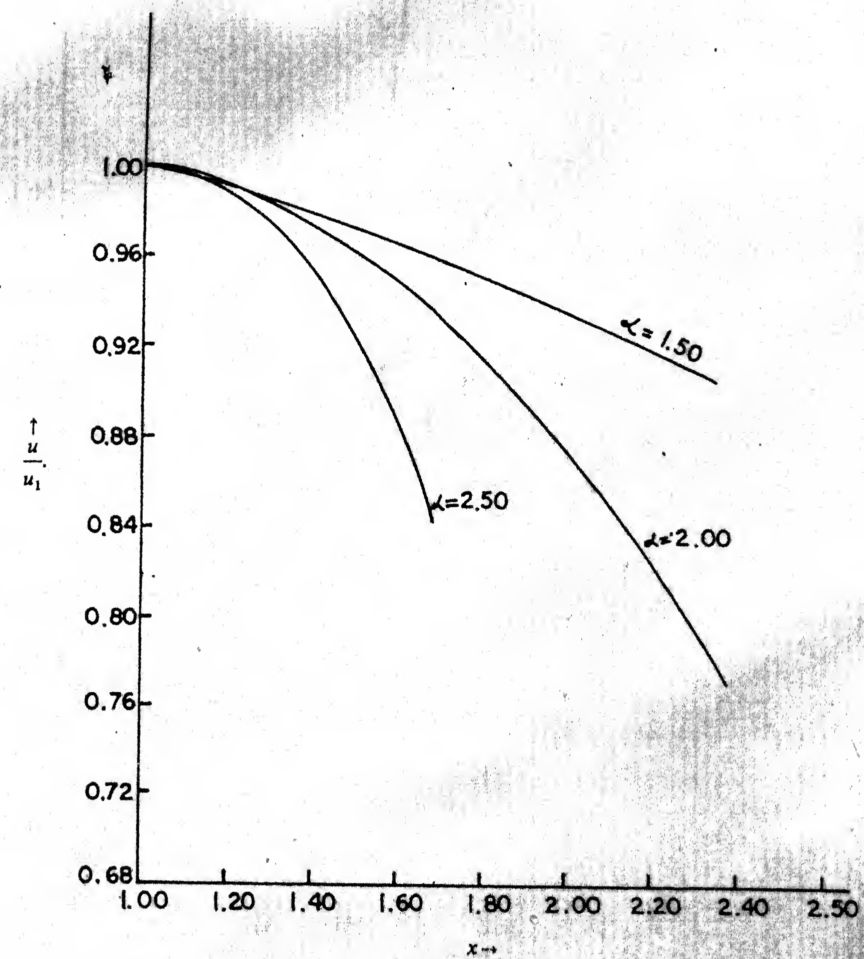


Fig. 1. Velocity distribution.

Using Equations (2.14) and (2.18) in Equations (2.6)–(2.9) and in Equation (2.13) we assume, without any loss of generality,

$$K = -\alpha\,,\quad \lambda = 0\,,\quad b = 4 + q\,,\quad a = -5 \quad \text{and} \quad \alpha = \frac{10}{4+q}\,. \tag{2.20}$$

Hence,

$$R = \eta_0^{-1/5}\, t^{(4+q)/5}\,, \tag{2.21}$$

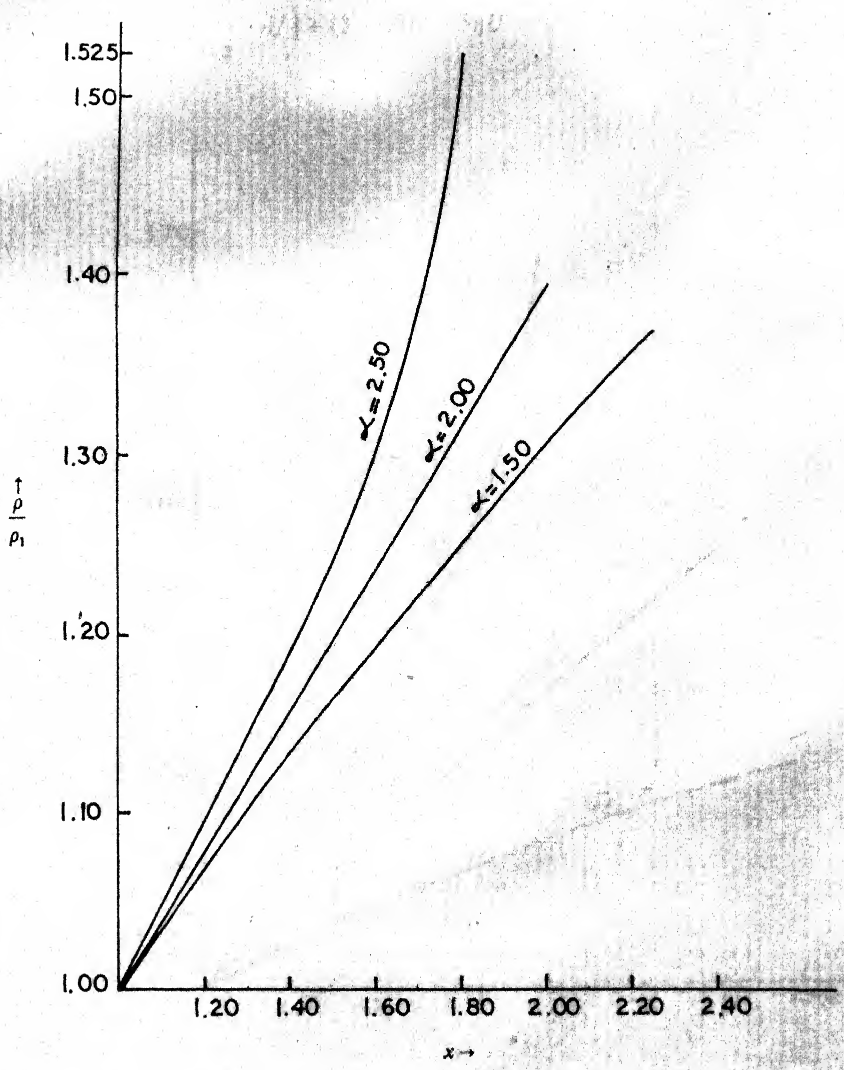


Fig. 2. Density distribution.

where it is assumed that initially $R = 0$. From Equation (2.21) it can be seen that the value $q = 1$ corresponds to uniform expansion of the sphere.

3. Solutions of the Problem

The condition inside the wave will be obtained from Equations (2.1)–(2.5). Using Equations (2.14), (2.15), and (2.20), the equations of motion are transformed to the following non-dimensional forms

$$5\eta\,\frac{\overline{\Omega}'}{\overline{\Omega}}\left(\frac{2}{\alpha} - U\right) + (3-\alpha)U - 5\eta U' = 0\,, \tag{3.1}$$

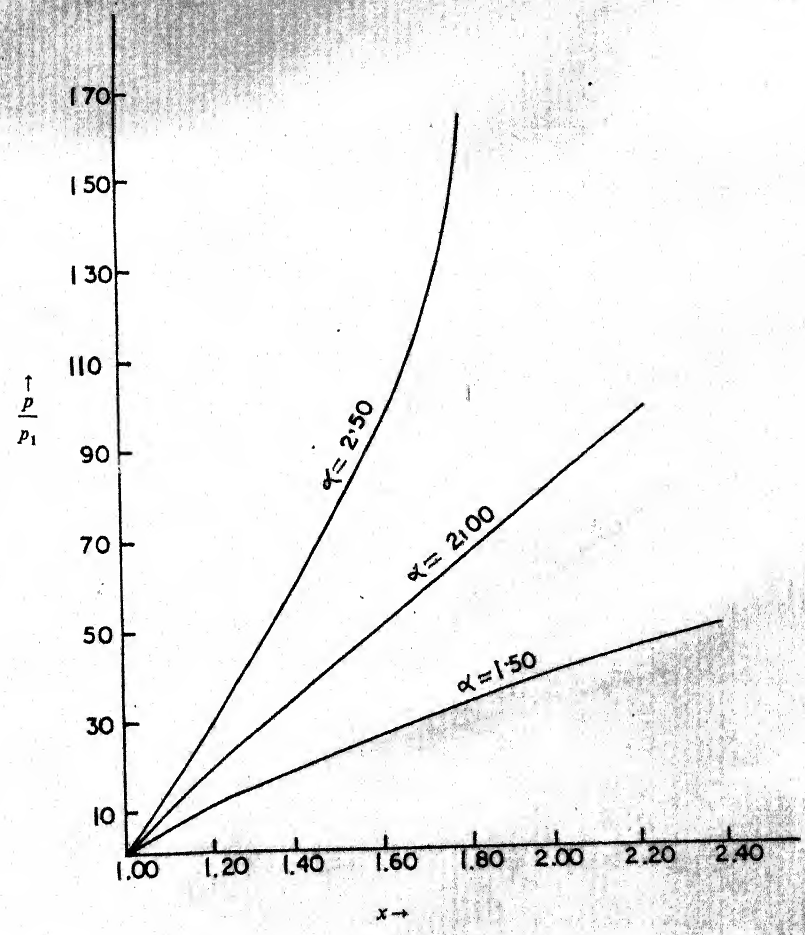


Fig. 3. Pressure distribution.

$$5U'\eta\left(\frac{2}{\alpha}-U\right)+U(U-1)+\frac{\overline{P}}{\overline{\overline{\Omega}}}(2-\alpha)-5\eta\,\frac{\overline{P}'}{\overline{\overline{\Omega}}}-\frac{\overline{N}^2}{\overline{\overline{\Omega}}}\left(\frac{\alpha-4}{2}+5\eta\,\frac{\overline{N}'}{\cancel{\overline{\overline{\Omega}}}\ \overline{N}}\right)+$$

$$+\frac{2(3-\alpha)(\alpha-1)}{\pi\alpha^2}\,\frac{1}{\eta^\alpha}\,L\overline{W}=0\,, \tag{3.2}$$

$$5\eta\,\frac{\overline{N}'}{\overline{N}}\left(\frac{2}{\alpha}-U\right)+2U-5\eta U'+\frac{U-1}{\cancel{2}}-\frac{1}{\cancel{A}_2}\,\alpha U=0\,, \tag{3.3}$$

$$5\eta\,\frac{\overline{P}'}{\overline{P}}\left(\frac{2}{\alpha}-U\right)+U(2-\alpha)-2-\gamma\left[-\alpha U+5\eta\,\frac{\overline{\Omega}'}{\overline{\overline{\Omega}}}\left(\frac{2}{\alpha}-U\right)\right]=0\,, \tag{3.4}$$

$$5\eta\,\frac{\overline{W}'}{\overline{W}}+4\pi\,\frac{\overline{\Omega}}{\overline{W}}-(3-\alpha)=0\,, \tag{3.5}$$

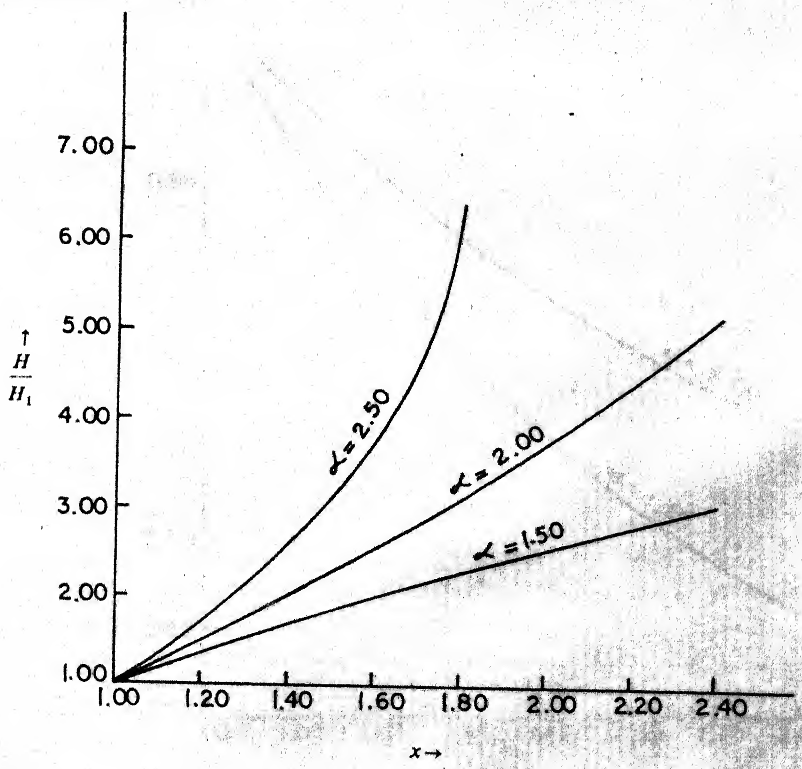


Fig. 4. Magnetic field distribution.

where

$$\bar{\Omega}' = \frac{\Omega'}{A}, \quad \bar{P}' = \frac{P'}{A}, \quad \bar{N}' = \frac{N'}{A}, \quad \bar{W}' = \frac{W'}{A},$$
$$\bar{\Omega} = \frac{\Omega}{A}, \quad \bar{P} = \frac{P}{A}, \quad \bar{N} = \frac{N}{A}, \quad \bar{W} = \frac{W}{A}, \tag{3.6}$$

$$L = \frac{1}{\gamma M^2} - \frac{2-\alpha}{2(\alpha-1)M_A^{+2}}; \tag{3.7}$$

and prime denotes the differentiation with respect to η. The jump conditions at the shock

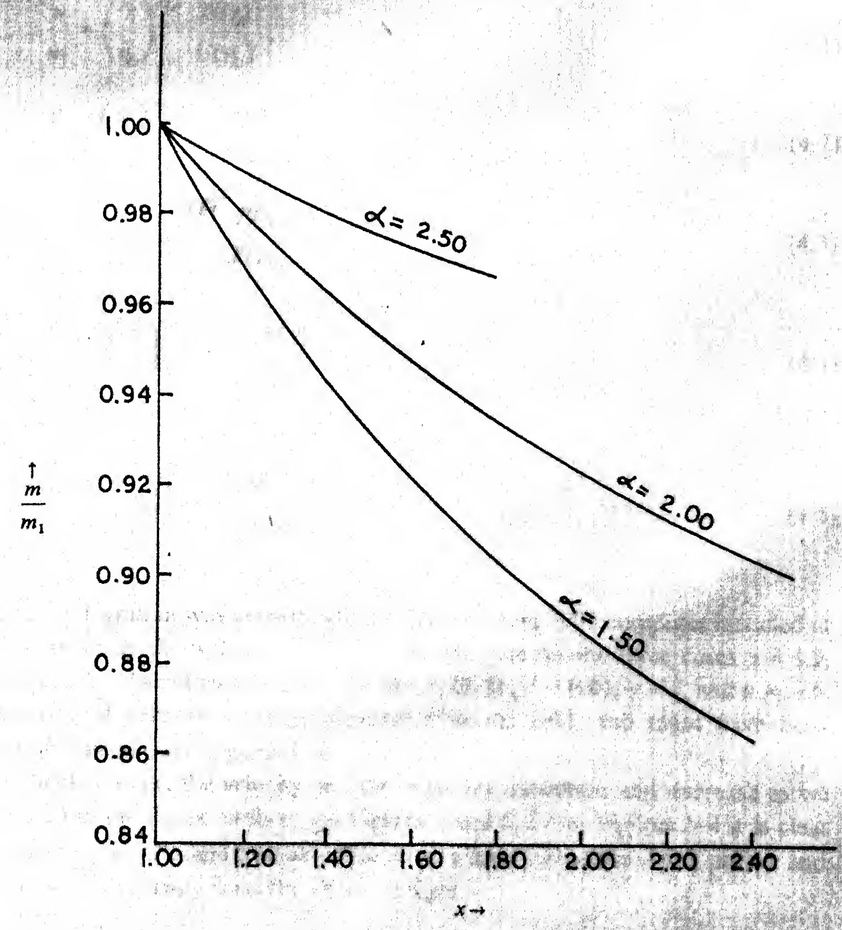


Fig. 5. Mass distribution.

are given by

$$U(\eta_0) = \frac{2}{\alpha}\,\frac{\xi - 1}{\xi}, \qquad \bar{\Omega}(\eta_0) = \xi, \qquad \bar{P}(\eta_0) = \frac{4}{\alpha^2}\,\psi,$$
$$\bar{N}(\eta_0) = \frac{2\xi}{\alpha}\,M_A^{-1} \quad \text{and} \quad \bar{W}(\eta_0) = \frac{4\pi}{3 - \alpha}. \tag{3.8}$$

4. Results and Discussion

Similarity solutions of the problem of propagation of a spherical magnetogasdynamic shock wave have been obtained. For numerical calculation the flow and field variables have been taken in the following non-dimensional forms:

$$\frac{u}{u_1} = \left(\frac{\eta}{\eta_0}\right)^{1/a} \frac{U(\eta)}{U(\eta_0)}, \tag{4.1}$$

$$\frac{\rho}{\rho_1} = \left(\frac{\eta}{\eta_0}\right)^{-\alpha/a} \frac{\Omega(\eta)}{\Omega(\eta_0)}; \tag{4.2}$$

$$\frac{p}{p_1} = \left(\frac{\eta}{\eta_0}\right)^{(2-\alpha)/a} \frac{P(\eta)}{P(\eta_0)}, \tag{4.3}$$

$$\frac{H}{H_1} = \left(\frac{\eta}{\eta_0}\right)^{(2-\alpha)/2a} \frac{N(\eta)}{N(\eta_0)} \tag{4.4}$$

and

$$\frac{m}{m_1} = \left(\frac{\eta}{\eta_0}\right)^{(3-\alpha)/a} \frac{W(\eta)}{W(\eta_0)}. \tag{4.5}$$

Numerical integration was performed on a DEC system 1090 computer installed at I.I.T., Kanpur using the well-known RKGS programme for the three cases $\alpha = 1.5$, $\alpha = 2$, and $\alpha = 2.5$. The other constants are $M^2 = 20$, $M_A^{-2} = 0.01$, $\gamma = \frac{5}{3}$, and $a = -5$. The variations of velocity, density, pressure magnetic field, and mass have been illustrated graphically, see Figures 1–5.

At the shock surface, the velocity and the mass are maximum and decrease as we move away from the shock surface, see Figures 1 and 5. From Figures 2–4 it is clear that the density, pressure, and magnetic field are at a minimum at the shock surface and increase as we move away from the shock surface.

Acknowledgement

One of the authors (V.K.S.) is thankful to Dr J. B. Singh, Department of Mathematics, SGR College, Dobi, for his encouragement in preparing this paper.

References

Carrus, P., Fox, P., Haas, F., and Kopal, Z.: 1951, *Astrophys. J.* **125**, 495.
Ryazanov, E. V.: 1959, *Soviet Phys. Dokl. SSSR* **4**, 537.
Sedov, L. I.: 1959, *Similarity and Dimensional Methods in Mechanics*, Academic Press, New York.
Singh, J. B.: 1982, *Astrophys. Space Sci.* **87**, 347.
Whitham, G. B.: 1958, *J. Fluid Mech.* **4**, 337.

A SELF-SIMILAR FLOW BEHIND AN EXPONENTIAL SHOCK WITH RADIATIVE HEAT FLUX, II

J. B. SINGH and P. R. VISHWAKARMA
Department of Mathematics, S.G.R. Postgraduate College, Dobhi, Jaunpur, India

(Received 2 February, 1983)

Abstract. Self-similar solutions for one-dimensional unsteady flow of a perfect gas behind an exponential shock propagating into an uniform atmosphere, are obtained here, taking radiation flux into consideration. The total energy of the wave is variable.

1. Introduction

Gusev (1957) and Ranga Rao and Ramana (1976) have studied the problem of unsteady self-similar motion of a gas displaced by a piston according to an exponential law. Verma and Singh (1979) and Singh and Srivastava (1982) have considered the problems of spherical shock waves in an exponentially increasing medium under the low uniform pressure including the effects of magnetic field and thermal radiation, respectively.

The present paper deals with a self-similar flow of a perfect gas behind an exponential shock driven out by an expanding surface, i.e., the surface of contact discontinuity which moves with time according to an exponential law. The shock waves propagate in an uniform atmosphere which is assumed to be at rest. The similarity solutions have been developed when the radiation heat flux is more important than the radiation pressure and radiation energy. The other assumptions include the gas to be grey and opaque and the shock to be transparent and isothermal. The total energy of the wave varies as the cube of the shock radius. Viscosity and the solar gravity have been neglected.

2. Equations of Motion and Boundary Conditions

The equations of flow behind a spherical shock are

$$\frac{d\rho}{dt} + \frac{\rho}{r^2}\frac{\partial}{\partial r}(ur^2) = 0, \tag{1}$$

$$\frac{du}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \tag{2}$$

$$\frac{dp}{dt} - \frac{\gamma p}{\rho}\frac{d\rho}{dt} + \frac{(\gamma - 1)}{r^2}\frac{\partial}{\partial r}(r^2 F) = 0, \tag{3}$$

$$p = \Gamma \rho T, \tag{4}$$

where u, ρ, p, and T are the velocity, density, material pressure and temperature, respectively; Γ being the gas constant and γ the ratio of specific heats while F denotes the heat flux.

Assuming local thermodynamic equilibrium, and taking Rosseland's diffusion approximation, we have

$$F = -\frac{c\mu}{3}\frac{\partial}{\partial r}(\sigma T^4), \tag{5}$$

where $\sigma c/4$ is the Stefan–Boltzmann constant, c the velocity of light, μ, the mean-free path of radiation, is a function of density and temperature. Following Wang (1966), we take

$$\mu = \mu_0 \rho^{\alpha} T^{\beta}, \tag{6}$$

μ_0, α, and β being constants.

The inner expanding surface moves with time according to an exponential law.

$$\bar{r} = A\exp(mt), \qquad (m > 0) \tag{7}$$

and since we have assumed self-similarity, the shock will also move with time according to an exponential law.

$$R = B\exp(mt), \tag{8}$$

where $\bar{r}$ is the radius of inner expanding surface, R is the shock radius, A, B, and m are dimensional constants.

The disturbance is headed by an isothermal shock; therefore, the boundary conditions are

$$u_1 = \left[1 - \frac{1}{\gamma M^2}\right] v, \tag{9}$$

$$\rho_1 = \gamma M^2 \rho_0, \tag{10}$$

$$p_1 = \rho_0 v^2, \tag{11}$$

$$F_1 = \frac{1}{2}\left[\frac{1}{\gamma M^4} - 1\right]\rho_0 v^3 \tag{12}$$

where subscripts 0 and 1 denote the regions immediately ahead and behind the shock front, respectively; and v is the shock velocity. M denotes the Mach number.

3. Similarity Solutions

The similarity transformations for the problem under consideration are:

$$\eta = \frac{r}{B\exp(mt)}, \tag{13}$$

$$u = vV(\eta), \tag{14}$$

$$\rho = \rho_0 G(\eta), \tag{15}$$

$$p = \rho_0 v^2 P(\eta), \tag{16}$$

$$F = \rho_0 v^3 Q(\eta). \tag{17}$$

The variable η assumes the value 1 at the shock and $\bar{\eta}$ on the inner expanding surface. This enables us to express the radius of the inner expanding surface $\bar{r} = \bar{\eta} R$.

Now using the Equations (4), (6), and (13)–(17) into the Equation (5), we get

$$Q = -N G^{(\alpha-\beta-4)} P^{(\beta+4)} \left[\frac{P'}{P} - \frac{G'}{G} \right] \tag{18}$$

with $\beta = -2$; α remaining arbitrary, $(0 \leq \alpha \leq 2)$, and

$$N = \frac{4mc\mu_0 \sigma \rho_0^{(\alpha-1)}}{3T^{(\beta+4)}} = \text{a dimensionless radiation parameter.}$$

Making use of the Equations (13)–(17), the Equations (1)–(3) are transformed into the forms

$$V' = \frac{2(VP/G) - \eta(\eta - V)[(G^{(1-\alpha)} Q/NP) - V]}{\eta[(\eta - V)^2 - (P/G)]}, \tag{19}$$

$$G' = \frac{G(\eta V' + 2V)}{\eta(\eta - V)}, \tag{20}$$

$$P' = G[(\eta - V) V' - V], \tag{21}$$

$$Q' = \langle G\eta[V'\{(\eta - V)^2 - (\gamma P/G)\} - V(\eta - V)] - 2P(\eta + \gamma V) - 2(\gamma - 1)Q \rangle / [\eta(\gamma - 1)], \tag{22}$$

where primes denotes differentiation with respect to η.

The appropriate transformed shock conditions are

$$V_{(1)} = \left[1 - \frac{1}{\gamma M^2} \right], \tag{23}$$

$$G_{(1)} = \gamma M^2, \tag{24}$$

$$P_{(1)} = 1, \tag{25}$$

$$Q_{(1)} = \frac{1}{2} \left[\frac{1}{\gamma M^2} - 1 \right]. \tag{26}$$

4. Numerical Solutions

For exhibiting the numerical solutions, it is convenient to write the flow variables in the non-dimensional forms as

$$\frac{u}{u_1} = \frac{V}{V_{(1)}}, \quad \frac{\rho}{\rho_1} = \frac{G}{G_{(1)}},$$

$$\frac{p}{p_1} = \frac{P}{P_{(1)}} \quad \text{and} \quad \frac{F}{F_1} = \frac{Q}{Q_{(1)}}. \tag{27}$$

TABLE 1
$N = 10$

η	u/u_1	ρ/ρ_1	p/p_1	F/F_1
1.00	1.0000	1.00000	1.00000	1.00000
0.98	1.02160	1.01921	0.97718	0.578678
0.96	1.04273	1.04060	0.96639	0.19186
0.94	1.06344	1.06421	0.96739	−0.16243
0.92	1.08377	1.09014	0.97969	−0.48817
0.90	1.10378	1.11848	1.00264	−0.79079
0.88	1.12352	1.14934	1.03558	−1.07671
0.86	1.14303	1.18288	1.07790	−1.35277
0.84	1.16235	1.12927	1.12914	−1.62588
0.82	1.18151	1.25873	1.18904	−1.90287
0.80	1.20052	1.30150	1.25752	−2.19037
0.78	1.21942	1.34789	1.33471	−2.49495
0.76	1.23819	1.39824	1.42095	−2.82125
0.74	1.25687	1.45293	1.51676	−3.18215
0.72	1.27543	1.51243	1.62288	−3.579023
0.70	1.29390	1.57725	1.74025	−4.021976
0.68	1.31227	1.64799	1.87005	−4.52014
0.66	1.33054	1.72537	2.01367	−5.08403
0.64	1.34870	1.81019	2.17280	−5.72591
0.62	1.36675	1.90339	2.34946	−6.46028
0.60	1.38468	2.00607	2.54601	−7.30450
0.58	1.40249	2.11955	2.76528	−8.27950
0.56	1.42017	2.24534	3.01065	−9.41077
0.54	1.43771	2.38528	3.28612	−10.72959
0.52	1.45511	2.54155	3.59653	−12.27459
0.50	1.47234	2.71676	3.94771	−14.09393
0.48	1.48940	2.914105	4.34674	−16.24810
0.46	1.506288	3.13746	4.80228	−18.81374
0.44	1.52296	3.39161	5.32502	−21.88883
0.42	1.53942	3.62851	5.92825	−25.59990
0.40	1.55565	4.01763	6.62867	−30.11209
0.38	1.57161	4.40646	7.44743	−35.64345
0.36	1.58729	4.86118	8.41170	−42.48563
0.34	1.60265	5.39767	9.55677	−51.034227
0.32	1.61766	6.036942	10.92913	−61.83454
0.30	1.63227	6.80719	12.59102	−75.65102
0.28	1.64645	7.747108	14.62720	−93.57737
0.26	1.66013	8.21080	17.15552	−117.21341
0.24	1.673241	10.37590	20.34351	−148.96023
0.22	1.68569	12.25699	24.43582	−192.52994
0.20	1.697380	14.72889	29.80097	−253.86424
0.18	1.70816	18.06907	37.01473	−342.87355
0.16	1.71786	22.73964	47.01767	−476.93058
0.14	1.72623	29.55685	61.43244	−688.4145
0.12	1.73294	40.07475	83.25880	−1042.5423
0.10	1.737484	57.55726	118.5668	−1684.7271
0.08	1.739091	89.85132	181.2596	−2987.7993
0.06	1.73664	159.9835	309.5294	−6126.2032
0.04	1.72688	361.9695	644.8004	−16303.104
0.02	1.70351	1467.9465	2160.4617	−80996.104
0.007	1.67116	9727.8308	10319.152	−639120.97

TABLE II
$N = 100$

η	u/u_1	ρ/ρ_1	p/p_1	F/F_1
1.00	1.00000	1.00000	1.00000	1.00000
0.98	1.02084	1.01997	0.99728	0.67336
0.96	1.04161	1.04171	0.99635	0.33083
0.94	1.06232	1.06533	0.99728	−0.02956
0.92	1.08297	1.09095	1.00017	−0.40831
0.90	1.10355	1.11871	1.00514	−0.80858
0.88	1.12407	1.14878	1.01231	−1.12321
0.86	1.14454	1.18133	1.02184	−1.68149
0.84	1.16494	1.21656	1.03391	−2.18413
0.82	1.18529	1.24571	1.04870	−2.66918
0.80	1.20559	1.29604	1.06643	−3.31444
0.78	1.22583	1.34085	1.08735	−3.79946
0.76	1.24601	1.38946	1.11173	−4.42914
0.74	1.26615	1.44228	1.13989	−5.10918
0.72	1.28623	1.49974	1.17217	−5.84617
0.70	1.30625	1.56233	1.20899	−6.64782
0.68	1.32623	1.63065	1.25079	−7.52315
0.66	1.34616	1.70535	1.29808	−8.48276
0.64	1.36603	1.78722	1.35147	−9.53912
0.62	1.38586	1.87714	1.41164	−10.7070
0.60	1.40563	1.97618	1.47938	−12.0040
0.58	1.42535	2.08555	1.55562	−13.4511
0.56	1.44502	2.20673	1.64144	−15.0735
0.54	1.46464	2.34143	1.73813	−16.9017
0.52	1.48420	2.49172	1.84719	−18.9727
0.50	1.50371	2.66008	1.97045	−21.3316
0.48	1.52317	2.84961	2.11007	−24.0344
0.46	1.54256	3.06367	2.26870	−27.1501
0.44	1.56190	3.30706	2.44956	−30.7657
0.42	1.58117	3.58529	2.65660	−34.9907
0.40	1.60038	3.90535	2.89475	−39.9654
0.38	1.61951	4.27613	3.17019	−45.8708
0.36	1.63857	4.70905	3.49075	−52.9440
0.34	1.65754	5.21892	3.86650	−61.4993
0.32	1.6743	5.82530	4.31055	71.9600
0.30	1.69522	6.55445	4.84021	84.9064
0.28	1.71390	7.44224	5.47879	101.1491
0.26	1.73246	8.53877	6.25823	121.845
0.24	1.75089	9.91573	7.22333	148.089
0.22	1.76917	11.7786	8.43858	−184.237
0.20	1.78728	13.9880	9.99963	232.471
0.18	1.80518	17.0799	12.0536	−299.860
0.16	1.82285	21.4299	14.83682	−397.429
0.14	1.84023	27.7259	18.7498	−545.143
0.12	1.85726	37.3923	24.5193	−781.998
0.10	1.87383	53.3690	33.5857	1192.179
0.08	1.88981	82.6822	49.1757	−1984.135
0.06	1.90493	145.832	79.9317	3789.765
0.04	1.91873	325.784	156.921	9287.3470
0.02	1.92980	1295.814	485.751	41556.363
0.007	1.93312	10565.264	2577.718	379451.04

The numerical integration is carried out until the kinematic condition $V_{(\eta)} = \eta$ is satisfied, and it has been performed on DES-System 1090 computer installed at the I.I.T., Kanpur by the well-known RKGS programme for $\gamma = 1.4$, $M^2 = 20$, $N = 10$, and 100, $\alpha = 1$. The nature of the field variables is illustrated in Tables I and II.

Acknowledgement

The author is thankful to Shri K. S. Singh for carefully performing the numerical integrations.

References

Gusev, V. N.: 1957, TSAGI Rep. BNI.
Ranga Rao, M. P. and Ramana, B. V.: 1976, *J. Math. Phys. Sci.* 10, 465.
Singh, J. B. and Srivastava, S. K.: 1982, *Astrophys. Space Sci.* 88, 277.
Verma, B. G. and Singh, J. B.: 1979, *Astrophys. Space Sci.* 63, 253.
Wang, K. C.: 1966, *Phys. Fluids* 9, 1922.